Applying dual numbers to closed-form examples

University of Texas at San Antonio

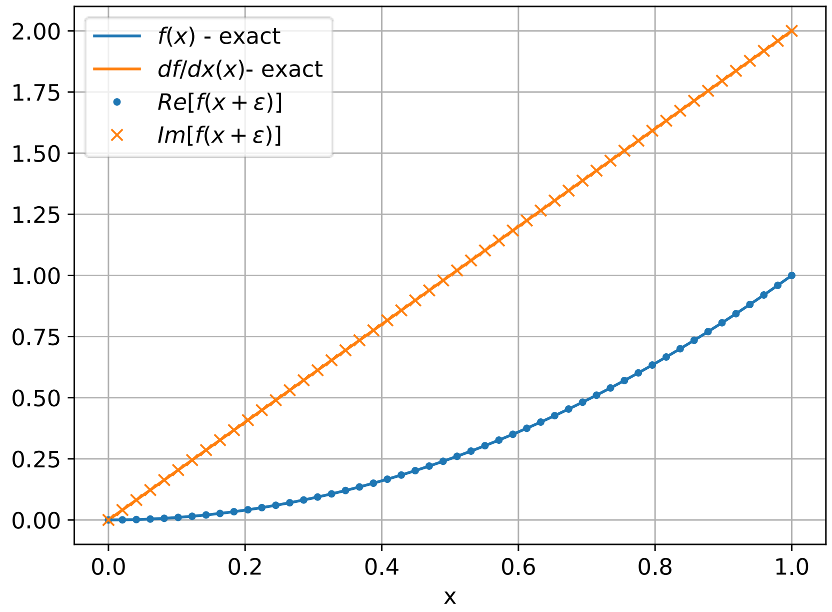
July, 2023

Closed-form example: $f(x) = x^2$

- The use of dual numbers to compute derivatives can be demonstrated using some closed-form examples. All examples assume $h = 1$.

$$f(x) = x^2$$
$$f(x+\epsilon) = (x+\epsilon)^2 = x^2 + 2x\epsilon$$

$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im((x+\epsilon)^2) = 2x$$

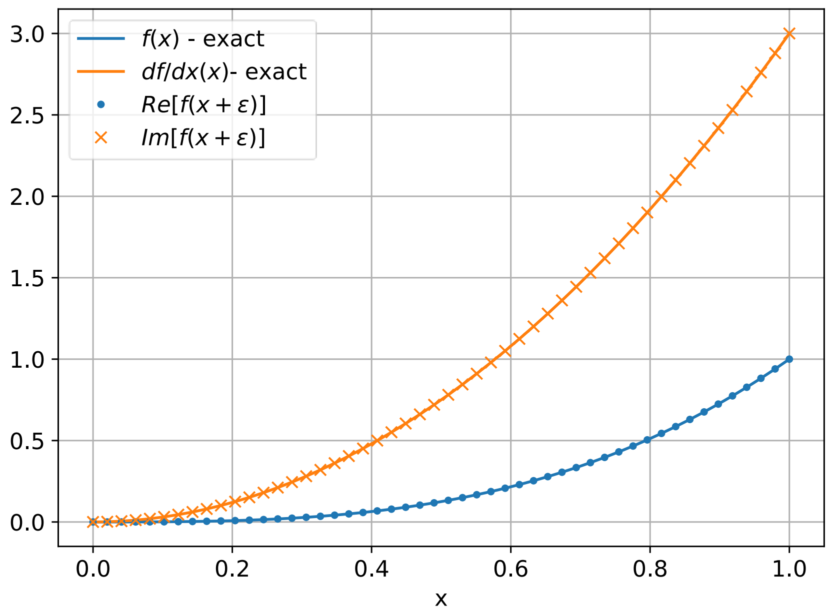


Closed-form example: $f(x) = x^3$

$$f(x) = x^3$$

$$f(x+\epsilon) = (x+\epsilon)^3 = x^3 + (3x^2)\epsilon + 3x\cancel{\epsilon^2}^{\,0} + \cancel{\epsilon^3}^{\,0}$$

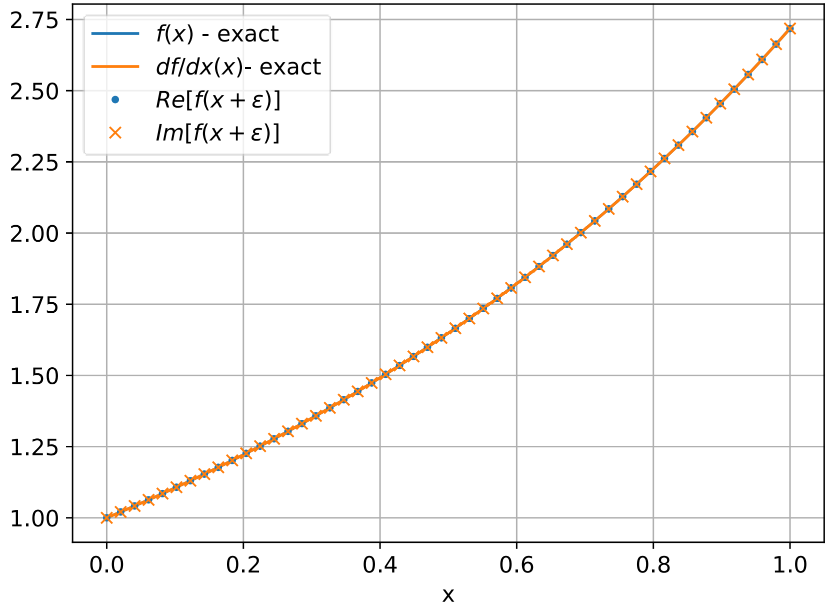
$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im((x+\epsilon)^3) = 3x^2$$



Closed-form example: $f(x) = e^x$

$$f(x) = e^x$$
$$f(x+\epsilon) = e^{x+\epsilon} = e^x + e^x\epsilon$$

$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im(e^{x+\epsilon}) = e^x$$

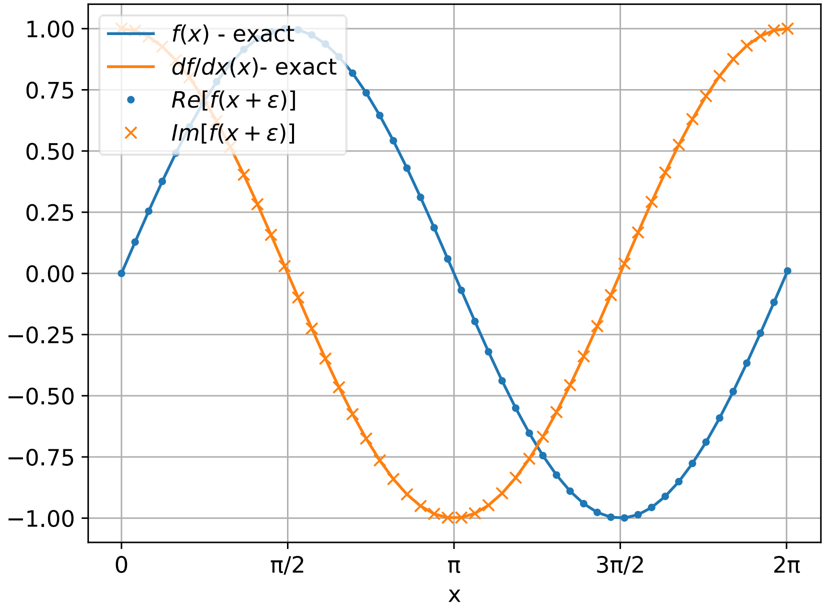


UTSA

Closed-form example $f(x) = \sin(x)$

$$f(x) = \sin(x)$$
$$f(x+\epsilon) = \sin(x+\epsilon) = \cos(x)\,\epsilon$$

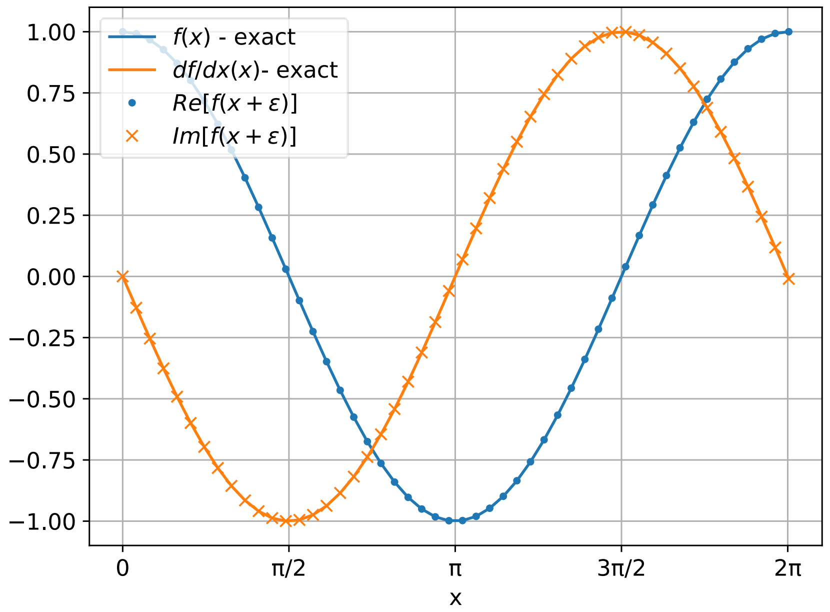
$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im(\sin(x+\epsilon)) \;= \cos(x)$$



Closed-form example: $f(x) = \cos(x)$

$$f(x) = \cos(x)$$
$$f(x+\epsilon) = \cos(x+\epsilon) = \cos(x) - \sin(x)\,\epsilon$$

$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im(\cos(x+\epsilon)) = -\sin(x)$$

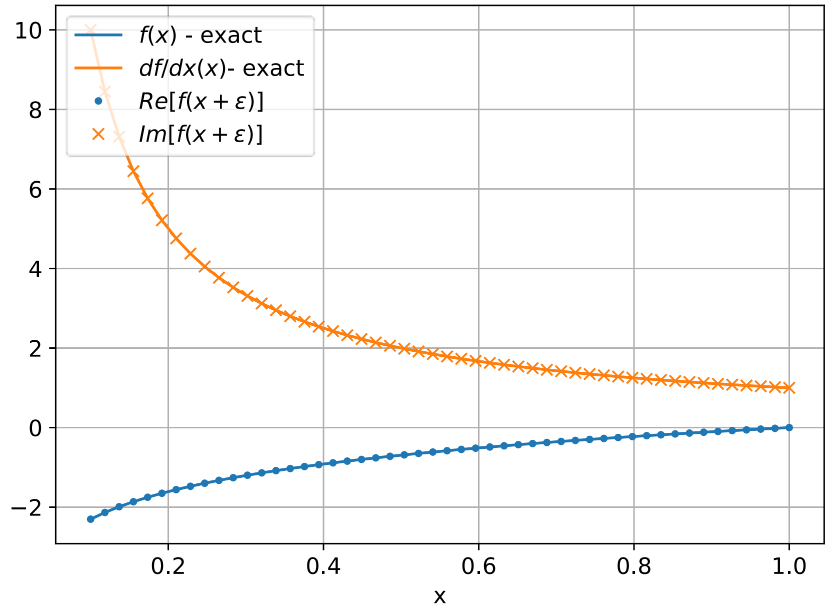


Closed-form example: $f(x) = \ln(x)$

$$f(x) = \ln(x)$$

$$f(x+\epsilon) = \ln(x+\epsilon) = \ln x + \frac{1}{x}\epsilon$$

$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im(\ln(x+\epsilon)) = \frac{1}{x}$$



Closed-form example: $f(x) = xe^{x}$

$$f(x) = xe^{x}$$

$$f(x+\epsilon) = (x+\epsilon)e^{x+\epsilon} = (x+\epsilon)(e^{x} + e^{x}\epsilon) =$$

$$xe^{x} + xe^{x}\epsilon + e^{x}\epsilon + \underbrace{e^{x}\epsilon^{2}}_{0}$$

$$xe^{x} + (xe^{x}\epsilon + e^{x}\epsilon) =$$

$$xe^{x} + (x+1)e^{x}\epsilon$$

$$\frac{df}{dx} = Im\big(f(x+\epsilon)\big) = Im\big((x+\epsilon)e^{x+\epsilon}\big) = (x+1)e^{x}$$