Assessing the accuracy of dual number differentiation using a step size study

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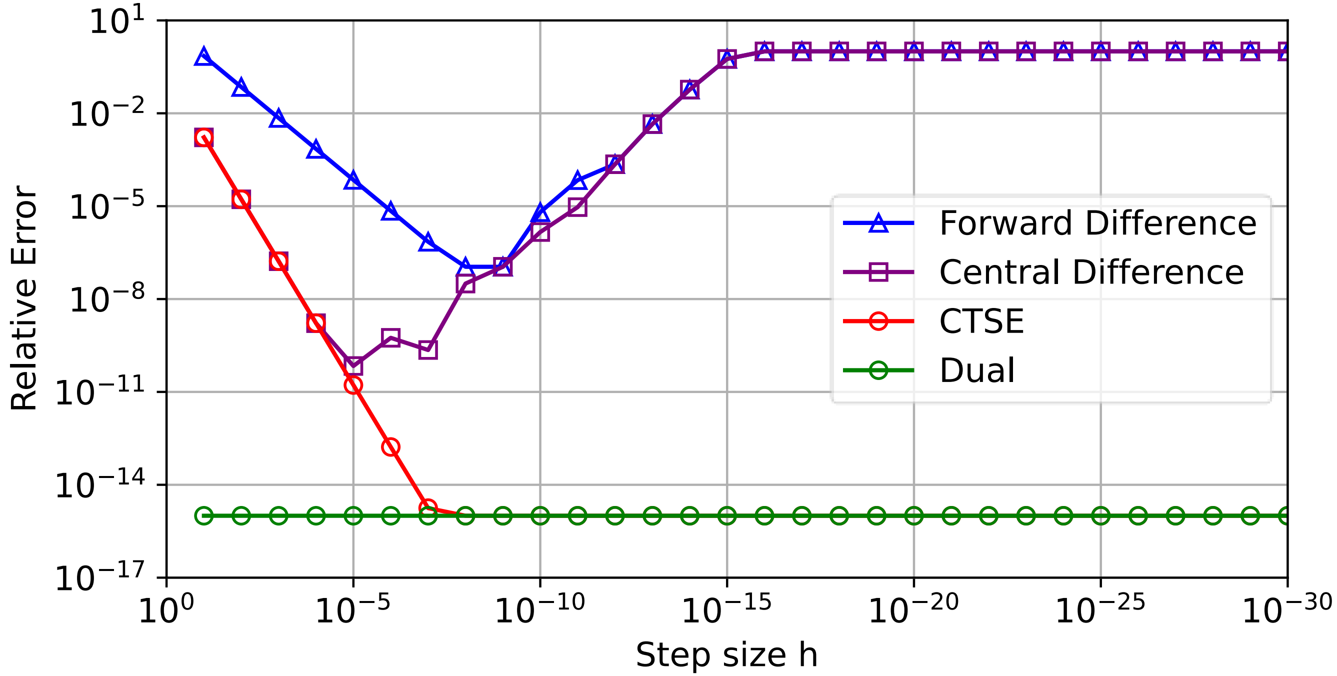
Step size study - $f(x) = sin(x)$

- Compare the accuracy of the dual number method against forward and central differencing.

$$f(x) = \sin(x)$$

$$f(1) = 0.84147098480790$$
$$f'(1) = 0.54030230586814$$



Forward: $\frac{df}{dx}(x) \approx \frac{f(x+h)-f(x)}{h}$

Central: $\frac{df}{dx}(x) \approx \frac{f(x+h)-f(x-h)}{2h}$

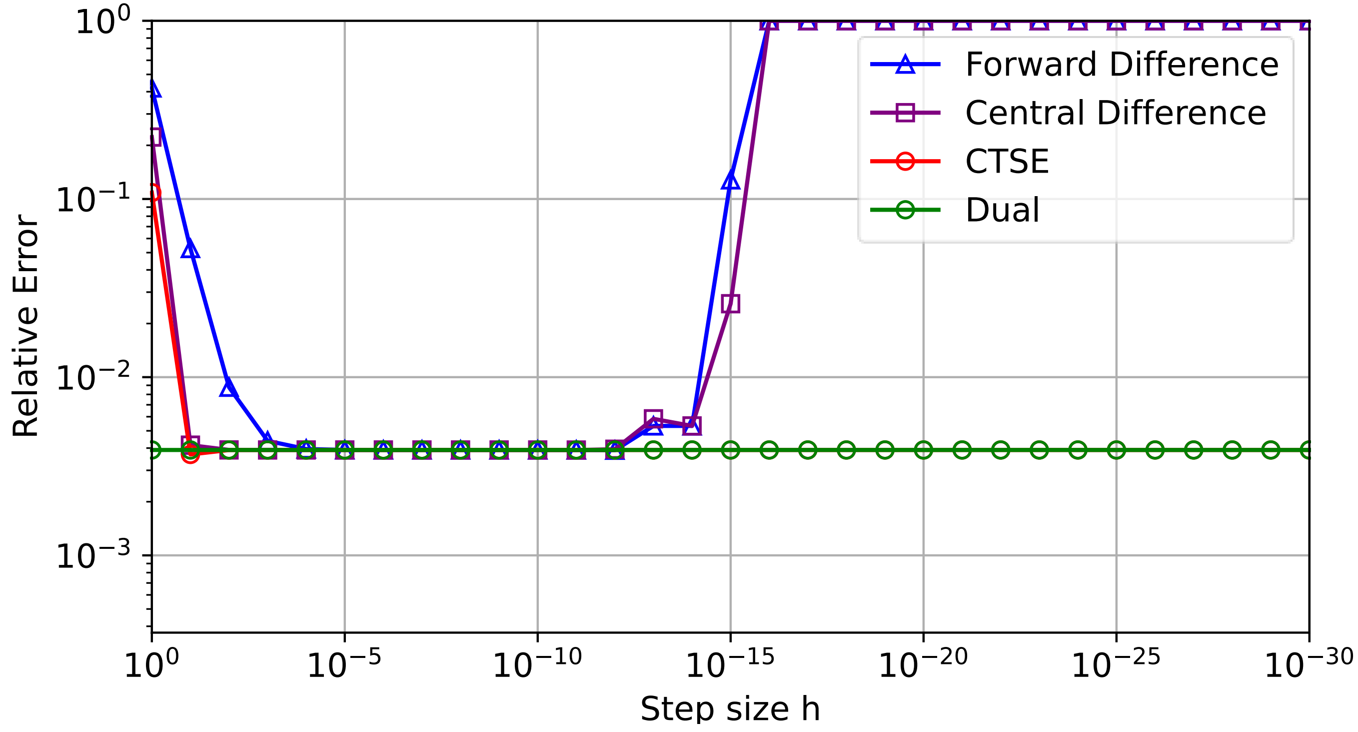
CTSE: $\frac{df}{dx}(x) \approx \frac{Im(f(x+ih))}{h}$

Dual: $\frac{df}{dx}(x) = Im(f(x+\epsilon))$

Step size study – numerical integration

- A similar behavior will be seen for more complicated algorithms; however, dual numbers will not always provide machine precision accuracy– *the accuracy is dependent upon the algorithm in which it is deployed*.

$$\frac{\partial}{\partial a} I(a,b,c) = \frac{\partial}{\partial a}\left(\int_a^b x e^{-cx} dx\right) \text{ with } a = 1, b = 2, c = 2$$



$\frac{\partial}{\partial a} I(a,b,c)$ using Simpson's rule