Automatic Differentiation Using Complex and Hypercomplex Variables

Assessing the accuracy of CTSE using a step size study

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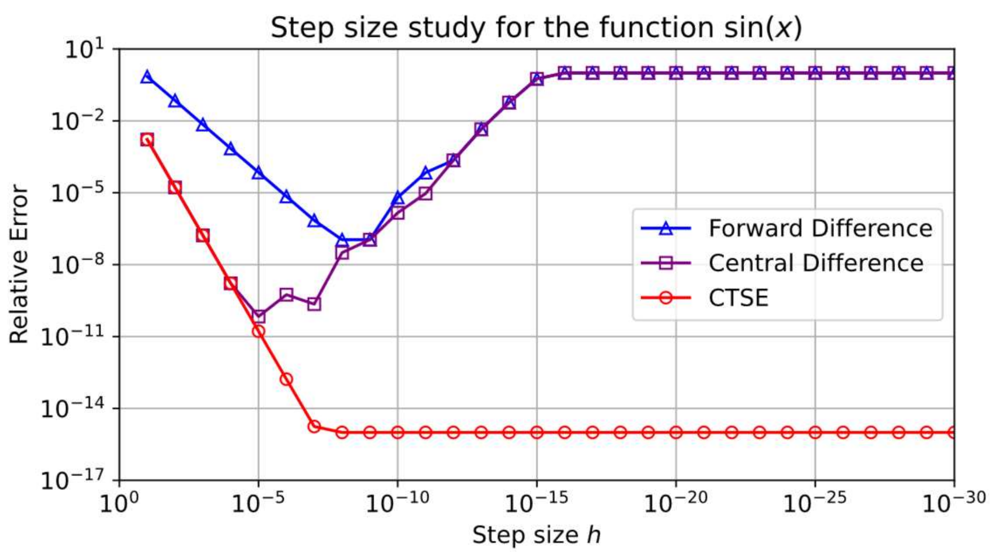
Step size study - $f(x) = sin(x)$

- CTSE converges as h^2 and produces machine precision results for $h < 10^{-8}$.
- FD converges as h with increasing error for $h < 10^{-9}$ due to subtractive error.
- CD converges as h^2 with increasing error for $h < 10^{-5}$ due to subtractive error.



CTSE: $\frac{df}{dx} \approx \frac{Im(f(x+ih))}{h}$

Forward: $\frac{df}{dx} \approx \frac{f(x+h)-f(x)}{h}$

Central: $\frac{df}{dx} \approx \frac{f(x+h)-f(x-h)}{2h}$

How small can we make h for CTSE?

- For a double precision system, h can be as small as 10^{-307}.

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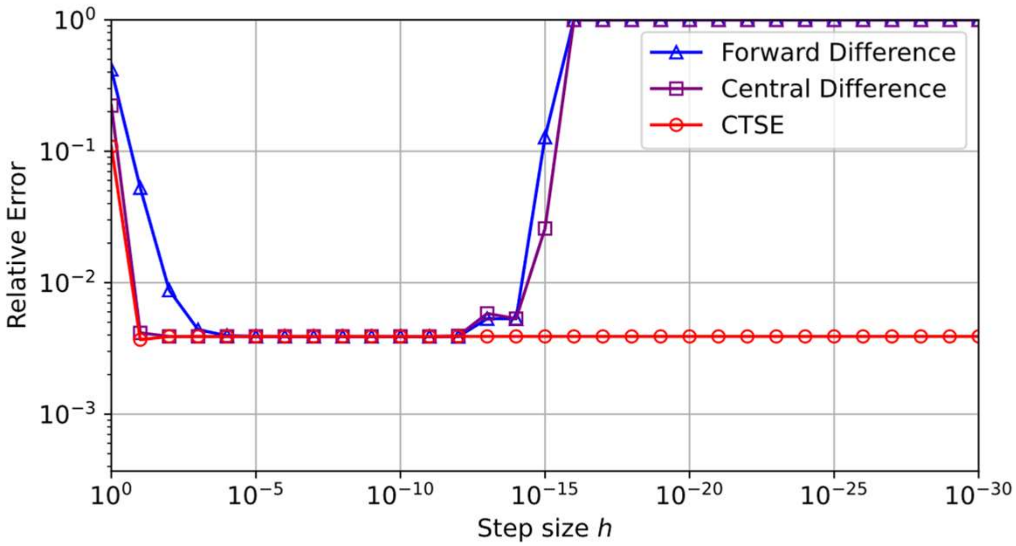
Step size study – numerical integration

- A similar behavior will be seen for more complicated algorithms; however, CTSE will not always provide machine precision accuracy– *the accuracy is dependent upon the algorithm in which it is deployed*.

$\frac{\partial}{\partial a} I(a,b,c) = \frac{\partial}{\partial a}\left(\int_a^b xe^{-c}\ dx\right)$ with $a = 1, b = 2, c = 2$



$\frac{\partial}{\partial a} I(a,b,c)$ using Simpson's rule

Note, CTSE results are independent of step size but are *NOT* machine precision accurate. The accuracy of Simpson's rule sets the accuracy of the differentiation.

How small can we make h for CTSE?

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