Applying dual numbers within a Newton-Rhapson solver

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July, 2023

Newton-Rhapson

- The Newton-Raphson Method is an algorithm that can be used to approximate the root of a function. The estimate of the root of function f at iteration $j + 1$ can be estimated from the result at step j as

$$x_{j+1} = x_j - \frac{f(x_j)}{\frac{df}{dx}(x_j)}$$

This equation can be rewritten to use dual numbers as (assuming $h = 1$)

$$x_{j+1} = x_j - \frac{Re\left(f\left(x_j + \epsilon\right)\right)}{Im\left(f\left(x_j + \epsilon\right)\right)}$$

Hence, one dual evaluation of $f\left(x_j + \epsilon\right)$ can be used to estimate $f(x_j)$ and $\frac{df}{dx}(x_j)$.

Newton-Rhapson

- Example1: $f(x) = \cos(ax) - x/a$

$$x_{j+1} = x_j - \frac{Re\left(\cos\left(a(x_j+\epsilon)\right)-(x_j+\epsilon)/a\right)}{\frac{1}{h}Im\left(\cos\left(a(x_j+\epsilon)\right)-(x_j+\epsilon)/a\right)}$$

Iteration no.	x value	Relative error
0	0.2	
1	0.850777122431116	$1.51 * 10^{-1}$
2	0.741530193469262	$3.31 * 10^{-3}$
3	0.739086449877212	$1.78 * 10^{-6}$
4	0.739085133215543	$5.18 * 10^{-13}$
5	0.739085133215161	0

Results for $a = 1$. Starting value $x = 0.2$. Assuming $h = 1$.