A novel method of filtration by the discrete heap transforms

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Outline

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Introduction: DsiHT

- We consider discrete unitary transforms, which we call *the heap transforms*, or transforms which are generated by the given signals.
- The complete systems of basis functions of heap transforms are referred to as waves generated by input signals, the waves with their specific motion in the space of functions.
- The heap transform is described by the unique set of angles, which represents the angular representation of the transform, or the signal-generator.

Introduction: DsiHT as a Linear Filter

- DFT is described by a beautiful but complex system of clockwise rotations of input data around an unique set of circles.
- **?** Other simple systems of rotations exist, which can be used in the signal and image processing.
- The heap transformation is described by the unique set of angles and represents a simple system of rotation.
- **!** When this system of rotations is performed over a signal with a component that is similar to the generator, this component is eliminated at each point except the first one where the energy of the component is moved.

Discrete Heap Transform: *Definition*

The *N*-point DsiHT by a generator $\mathbf{x}=(x_0,x_1,x_2,\ldots.x_{N-1})$ is defined by the following composition of basic transforms

$$T = T_{\varphi_1,\ldots,\varphi_m} = T_{\varphi_{i(m)}} \ldots T_{\varphi_{i(2)}} T_{\varphi_{i(1)}}$$

where *i(k)* is the permutation of numbers $k=1,2,\ldots,m$.

We consider the case when each transformation T_{φ_k} changes only two components of the input vector

$$z=(z_1,z_2,\ldots,z_{N-1})'$$

The transform T_{φ_k} is represented as

$$T_{\varphi_k} : \mathbf{z} \rightarrow \left(z_1, \ldots, z_{k_1-1}, \underline{f_{k_1}(\mathbf{z},\varphi_k)}, z_{k_1+1}, \ldots, z_{k_2-1}, \underline{f_{k_2}(\mathbf{z},\varphi_k)}, z_{k_2+1}, \ldots, z_m\right)$$

DsiHT: *System of decision equations*

In the transform

$$T_{\varphi_k} : \mathbf{z} \to (z_1, ..., z_{k_1-1}, \underline{f_{k_1}(\mathbf{z}, \varphi_k)}, z_{k_1+1}, ..., z_{k_2-1}, \underline{f_{k_2}(\mathbf{z}, \varphi_k)}, z_{k_2+1}, ..., z_m)$$

The pair of numbers (k_1,k_2) is uniquely defined by k, and the operation $k \longrightarrow (k_1,k_2)$ defines the path of the transform.

- Assume that such two functions f and g exists that

$$T_{\varphi_k} = T_{k_1,k_2}(\varphi_k) : (z_{k_1}, z_{k_2}) \to (f(z_{k_1}, z_{k_2}, \varphi_k), g(z_{k_1}, z_{k_2}, \varphi_k)).$$

a. $f(x,y,\varphi)$ and $g(x,y,\varphi)$ are functions of three variables.

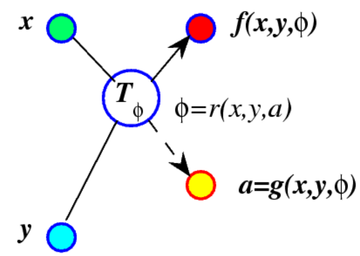
b. φ is referred to as the rotation parameter such as the angle, and x and y as the coordinates of the point (x,y) on the plane.

DsiHT: Basic Transformations

The selection of $\{\varphi_k\}$ is initiated by the vector-generator through the so-called decision equations and a given set of constants $\boldsymbol{A} = \{a_1, a_2, \ldots, a_{N-1}\}$ in the following way.

The system of equations

$$\begin{cases} f(x, y, \varphi) = y_0 \\ g(x, y, \varphi) = a \end{cases}$$



is called *the system of decision equations*.

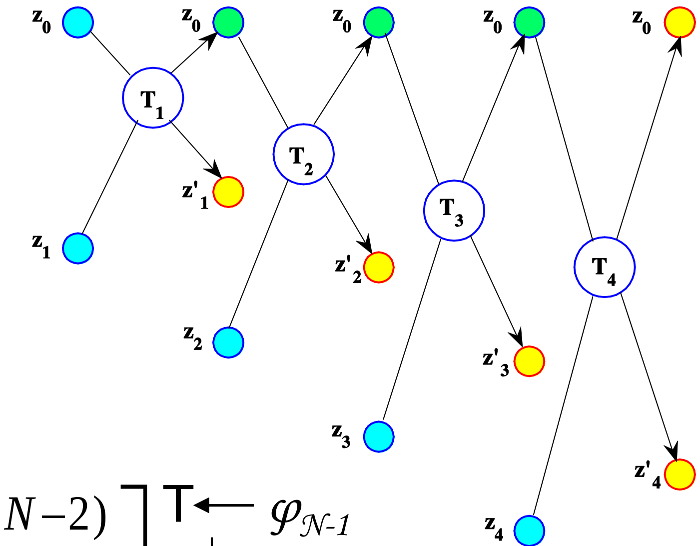
1. The value of φ is calculated from the second equation which is called *the angular equation*.
2. The value of y_0 is calculated from the given input (x,y) and obtained φ.

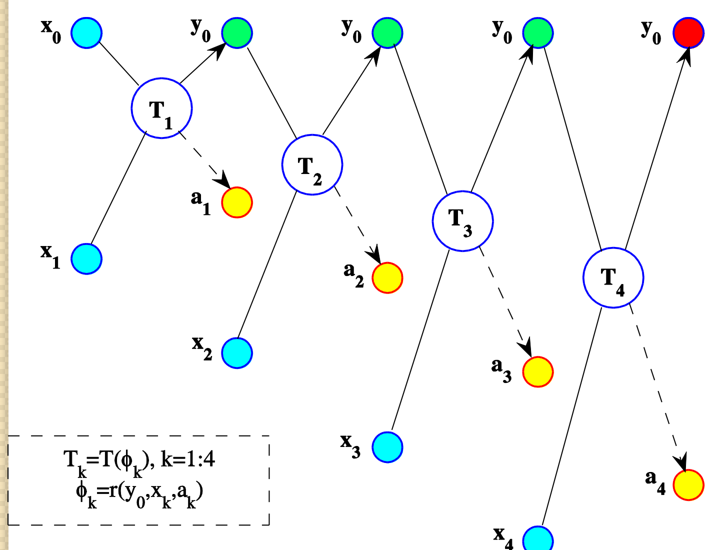
DsiHT: Composition and Application

$$\begin{bmatrix} x_0 \\ x_1 \end{bmatrix}^{T} \leftarrow \varphi_1 \quad \rightarrow \quad \begin{bmatrix} y_0^{(1)} \\ a_1 \end{bmatrix}^{T} \leftarrow \varphi_2 \quad \rightarrow \quad \begin{bmatrix} y_0^{(1)} \\ x_2 \end{bmatrix} \quad \cdots$$

$$\begin{bmatrix} y_0^{(N-2)} \\ x_{N-1} \end{bmatrix}^{T} \leftarrow \varphi_{\mathcal{N}-1} \quad \rightarrow \quad \begin{bmatrix} y_0^{(N-1)} \\ a_{N-1} \end{bmatrix}$$

$$z \rightarrow H(z):$$





x-generator is processed first and during this process all angles φ_k are calculated

Coordinated network of the DsiHT

Case: A $= \{a_1, a_2, \ldots, a_{N-1}\} = \{0, 0, \ldots, 0\}$

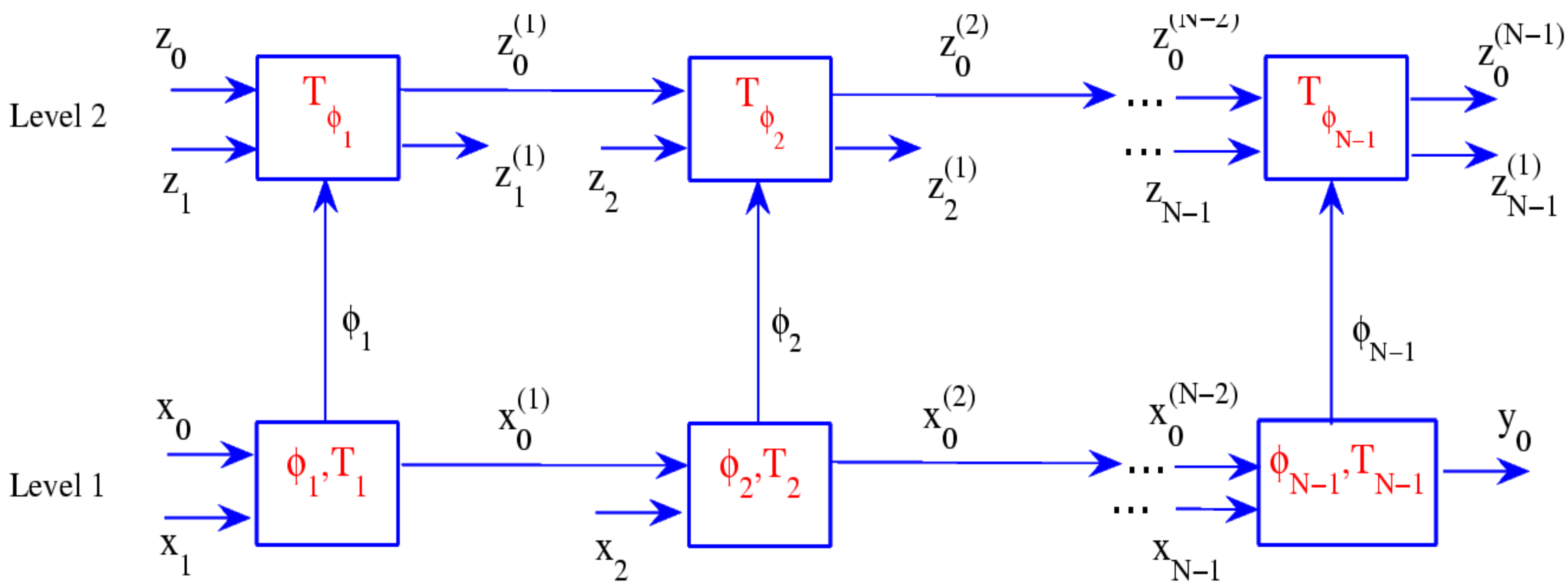


Fig. 1. Network of the ***x***-induced DsiHT of the signal ***z***.

DsiHT: *Example 1. Elementary rotations*

Given a real number a consider the following functions defined on the set of points $\{(x,y);\ x^2+y^2 \geq a^2\}$

$$\begin{cases} f(x,y,\varphi) = x\cos\varphi - y\sin\varphi, \\ g(x,y,\varphi) = x\sin\varphi + y\cos\varphi. \end{cases}$$

It is a rotation of the point (x,y) to the horizontal $Y=a$,

$$T_\varphi:\ (x,y) \rightarrow (y_0,a) = (x\cos\varphi\ -\ y\sin\varphi, a).$$

$$T_\varphi : \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} y_0 \\ a \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix},$$

$$\varphi = \arccos\left(\frac{a}{\sqrt{x^2+y^2}}\right) - \arctan\left(\frac{x}{y}\right), \qquad \left(\varphi = \arccos\left(\frac{a}{x}\right) \text{if } \ y = 0\right).$$

DsiHT: *Example N=8,* $x=(1,1,-1,-1,1,1,-1,-1)'$

Generator is transformed into the scaled unit vector

$$T(x) = \| x \| e_1 = (\| x \|, 0, 0, \ldots, 0)' = (\sqrt{8}, 0, 0, \ldots, 0)'$$

$$\mathbf{T} = \mathrm{diag}\left\{\begin{matrix} 0.3536 \\ 0.7071 \\ 0.4082 \\ 0.2887 \\ 0.2236 \\ 0.1826 \\ 0.1543 \\ 0.1336 \end{matrix}\right\} \cdot \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 3 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & 4 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & -1 & 5 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 1 & 6 & 0 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & 7 \end{bmatrix}$$

The angels of rotations are

$$\{-0.7854, 0.6155, 0.5236, -0.4636, -0.4205, 0.3876, 0.3614\}$$

DsiHT: *Example N=8, x=(1,1,-1,-1,1,1,-1,-1)'.*

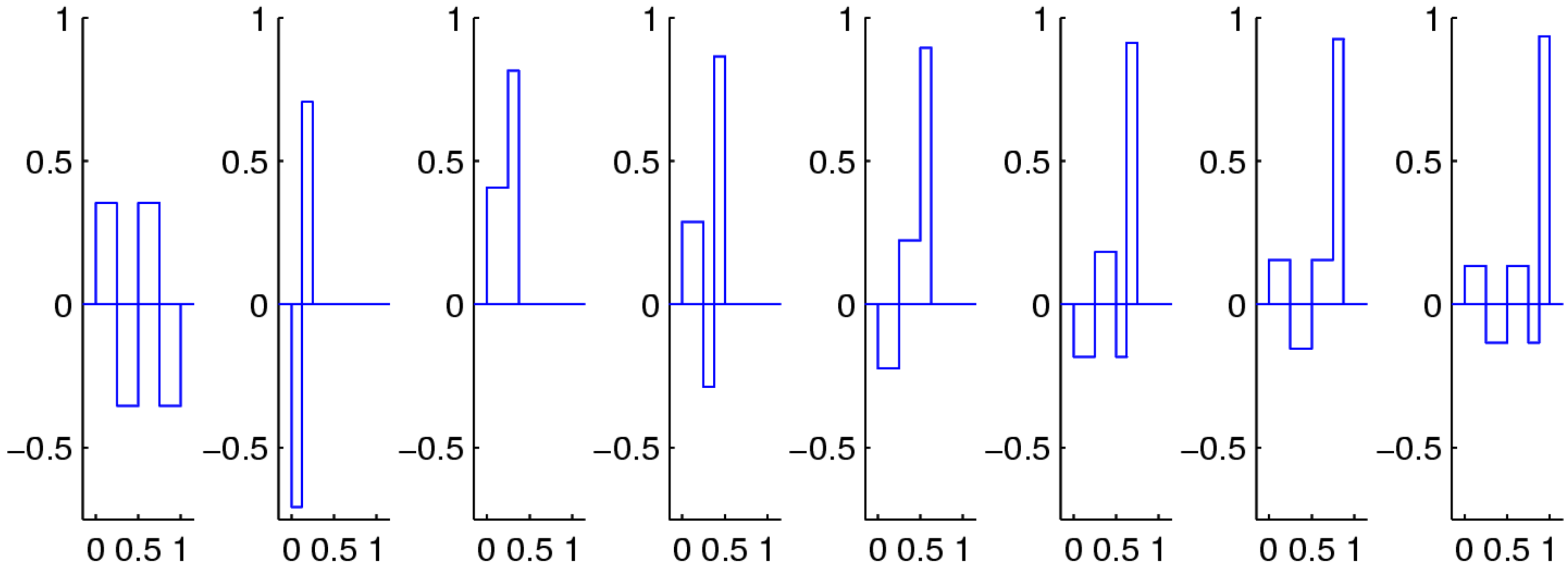


Fig. 2. Basis waves of the 8-point DsiHT.

$$m_4 = [1, 1, -1, 3, 0, ..., 0]$$
$$m_2 = [-1, 1, 0, 0, ..., 0] \qquad \hat{m}_2 = [0, 0, -1, 1, 0, ..., 0]$$
$$m_3 = [1, 1, 2, 0, ..., 0] \qquad m_4 = m_3 + 3\hat{m}_2$$

Properties of the DsiHT

$$T(\mathbf{x}) = (||\mathbf{x}||, 0, 0, \ldots, 0)',$$
$$T(\mathbf{z}) = (z_0^{(N-1)}, z_1^{(1)}, z_2^{(1)}, \ldots z_{N-1}^{(1)})'.$$

Consider the discrete-time signal of length 512 sampled from

$$z(t) = 2cos(2t) – 4sin(16t), \ t \in [0,2\pi].$$

- The DsiHT generated by cos(2t) will removes from the signal z(t) the similar wave.
- The DsiHT generated by sin(16t) will removes from the signal z(t) the similar wave.

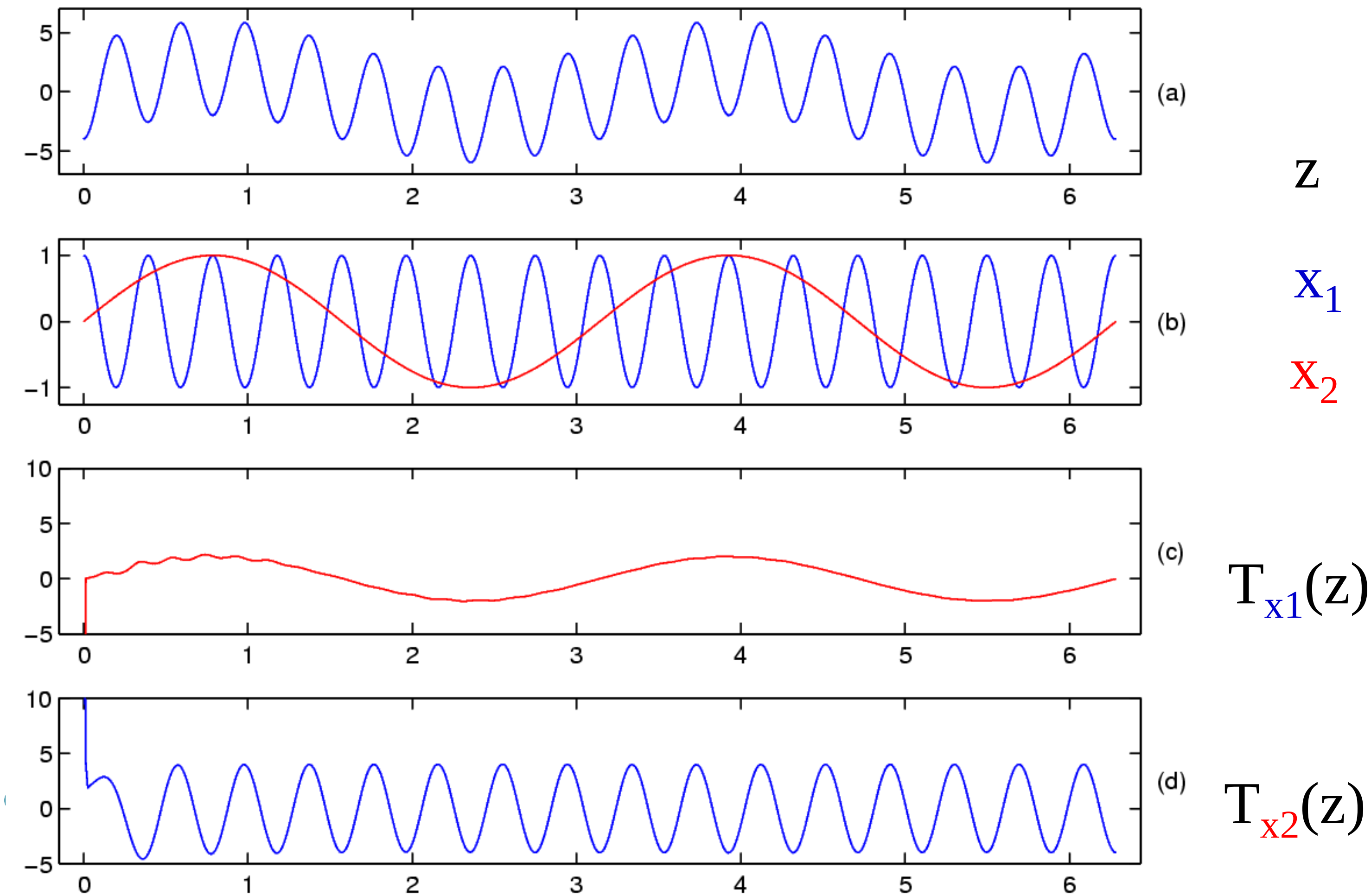


Fig. 3. (a) 512-point discrete signal **z**, (b) generators $\mathbf{x}_1$ and $\mathbf{x}_2$, and (c),(d) the 512-point $\mathbf{x}_1$- and $\mathbf{x}_2$-generated DHTs.

Method of the DFT

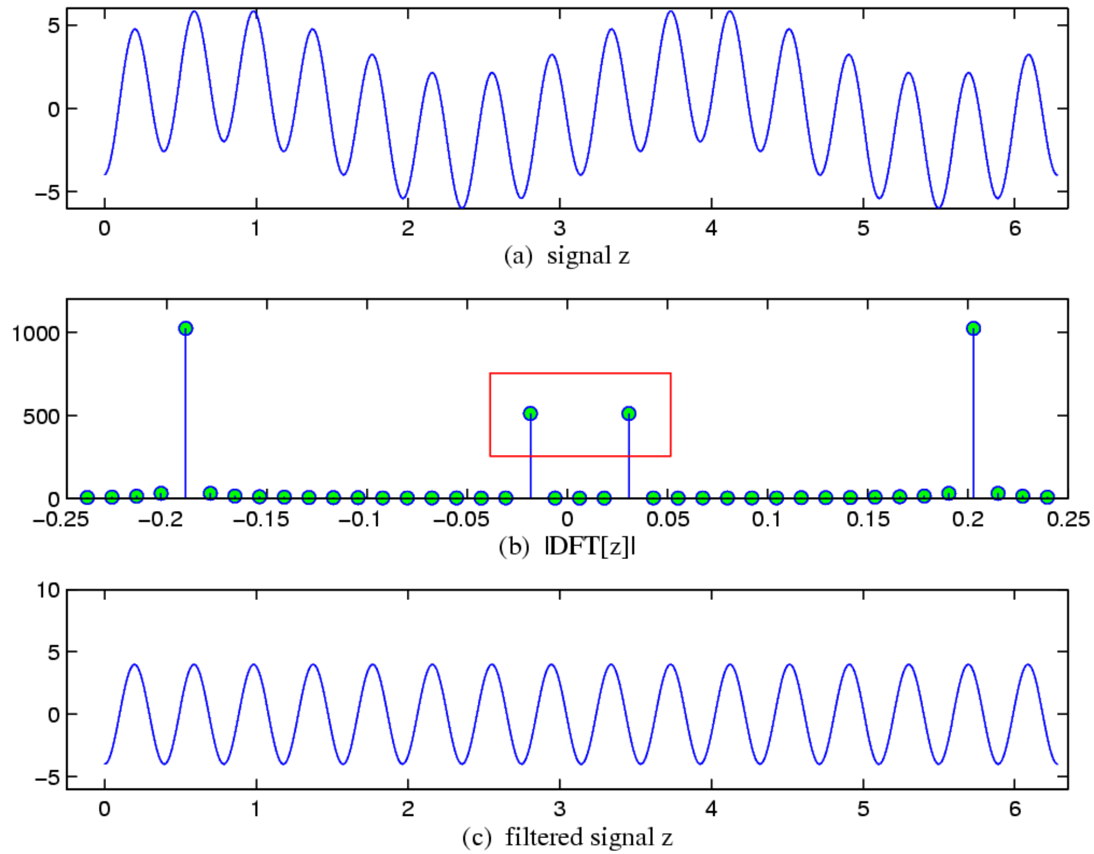


Fig. 4. (a) 512-point discrete signal **z**, (b) magnitude of the DFT of the signal, and (c) the filtered signal.

DsiHT and noisy signal *y(t)=x(t)+n(t)*

$$n(t) = \cos(\omega_1 t) + 3\sin(\omega_2 t) + 5\sin(\omega_3 t)$$

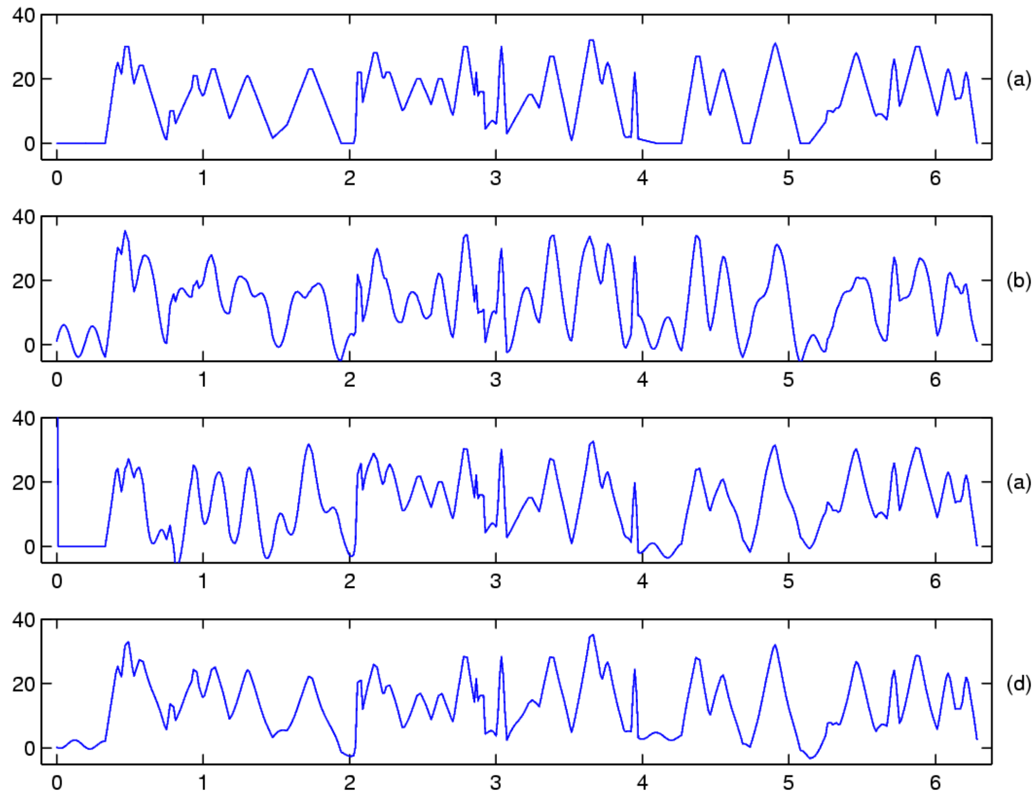


Fig. 5. (a) Original signal, (b) noisy signal, (c) filtered signal with DsiHT, and (d) filtered signal with DFT.

THE FILTRATION OF 2-D IMAGES

(a) (b) (c)

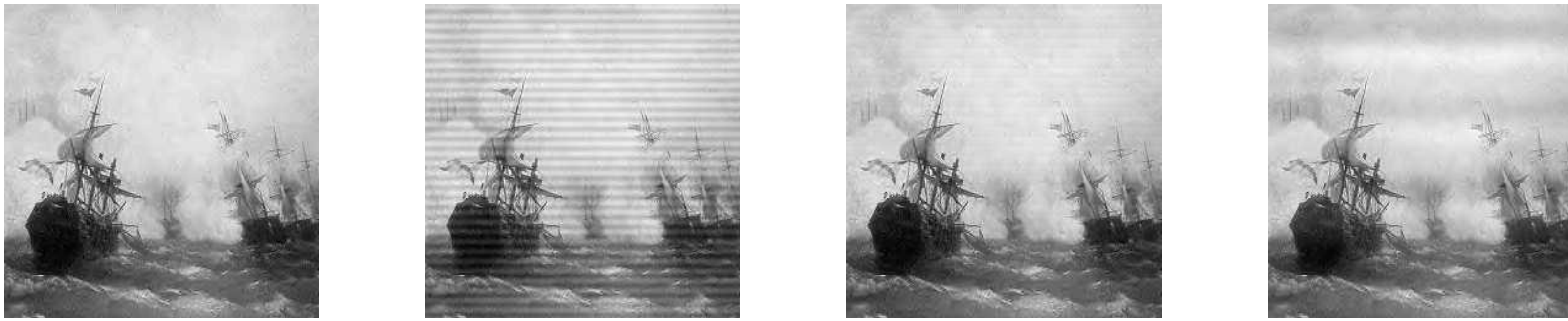
Fig. 6. (a) The original image, (b) the image corrupted with sin(64t) along the columns, and (c) filtered image by the sin(64t)-induced heap transform

THE FILTRATION OF 2-D IMAGES

(a) (b) (c)

Fig. 7. (a) The tree image, (b) the image corrupted with sin(128t) along the columns, and (c) filtered image by the sin(128t)-induced heap transform.

FILTRATION OF 2-D IMAGES



(a) (b) (c) (d)

Fig. 8. (a) The Aivazowsky's image, (b) the image corrupted with sin(64t) along the rows, (c) filtered image by the sin(64t)-induced heap transform, and (d) filtered image by the sin(8t)-induced heap transform over the image corrupted with sin(8t) along the rows

FILTRATION OF 2-D IMAGES

(a) (b) (c)

Fig. 9. (a) The Aivazowsky's image, (b) the image corrupted with a Gaussian noise, and (c) the detected Gaussian noise from the corrupted image with the help of DsiHT (the noise image has been scaled).

THE FILTRATION OF 2-D IMAGES

- When an image is corrupted with noise, the noisy points in image normally have values much higher or lower than the image value.

  If each row/column of the image is considered as the input signal x and the median of that row/column is considered as the vector-generator z of the heap transform, the remaining image is enhanced version of the original image.

ENHANCEMENT OF 2-D IMAGES

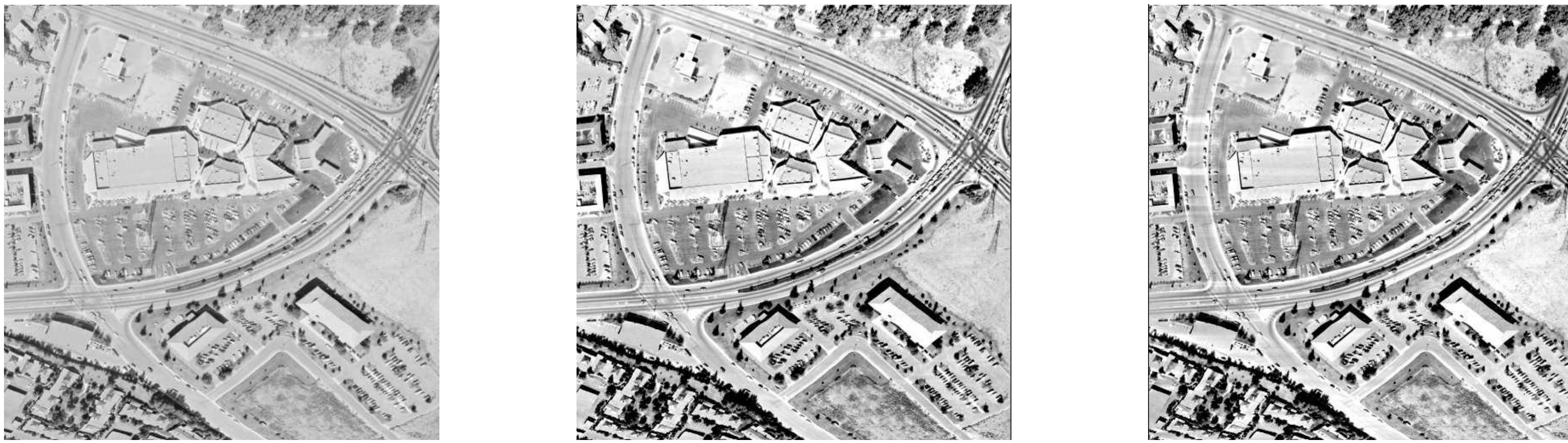
(a)

(b)

(c)

- Fig. 10. (a) The chemical plant image, EME=12.47, (b) the enhanced image with histogram equalization, EME=20.21,and (c) the enhanced image by the heap transform, EME=36.40

ENHANCEMENT OF 2-D IMAGES



(a) (b) (c)

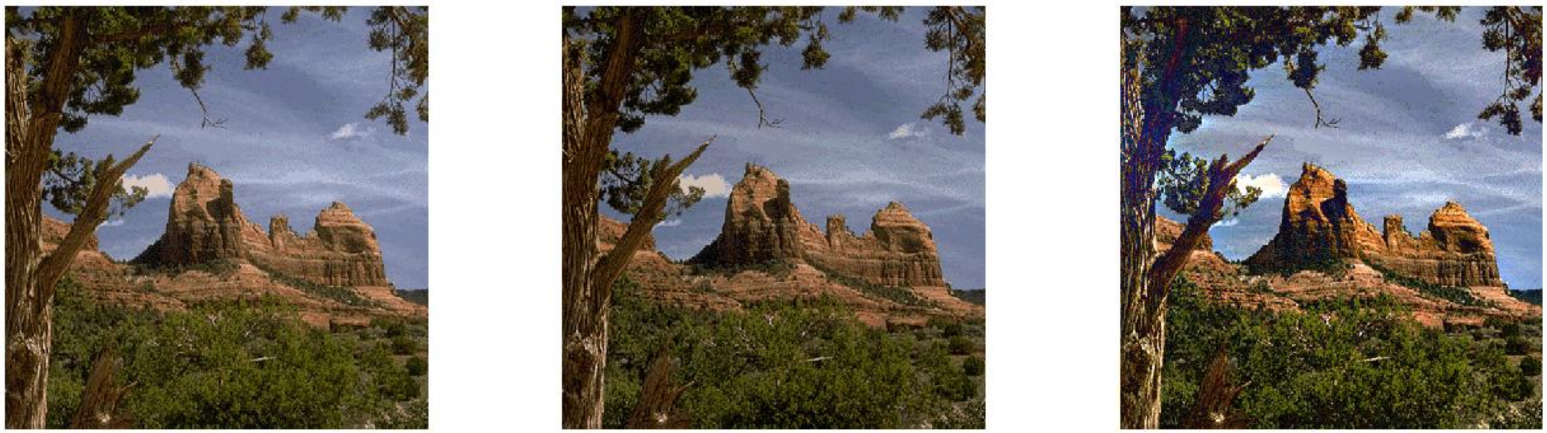
Fig. 11. (a) The original gray-scale image, EME=8.85, (b) the enhanced image with the natural path heap transform, EME=25.27, and (c) the enhanced image with the strong path heap transform, EME=25.03

ENHANCEMENT OF 2-D IMAGES

(a) (b) (c)

Fig. 12. (a) The color image, average EME=13.04, (b) the enhanced image with α-rooting method (the average EME is 19.30 and α=0.90 for all color channels), and (c) the enhanced image by the heap transform (the average EME is 31.63).

ENHANCEMENT OF 2-D IMAGES



(a) (b) (c)

Fig. 13. (a) The color image (average EME is 17.75), (b) enhanced image by the α-rooting method (average EME is 18.98 and α =0.98 for all color channels), and (c) enhanced image by the heap transform (average EME is 59.00).

ENHANCEMENT OF 2-D IMAGES

(a) (b) (c)

Fig. 14. (a) The image (average EME is 16.50), (b) enhanced image by the α-rooting method (average EME is 20.43 and α =0.90,0.98,0.98 for three color channels), and (c) image enhanced by the heap transform (average EME is 43.6).

FILTRATION OF 2-D IMAGES

(a) (b) (c)

Fig. 15. (a) The Barbara image, (b) the image corrupted with sin(64t) along the columns, and (c) filtered image by the sin(64t)-induced heap transform.

FILTRATION OF 2-D IMAGES

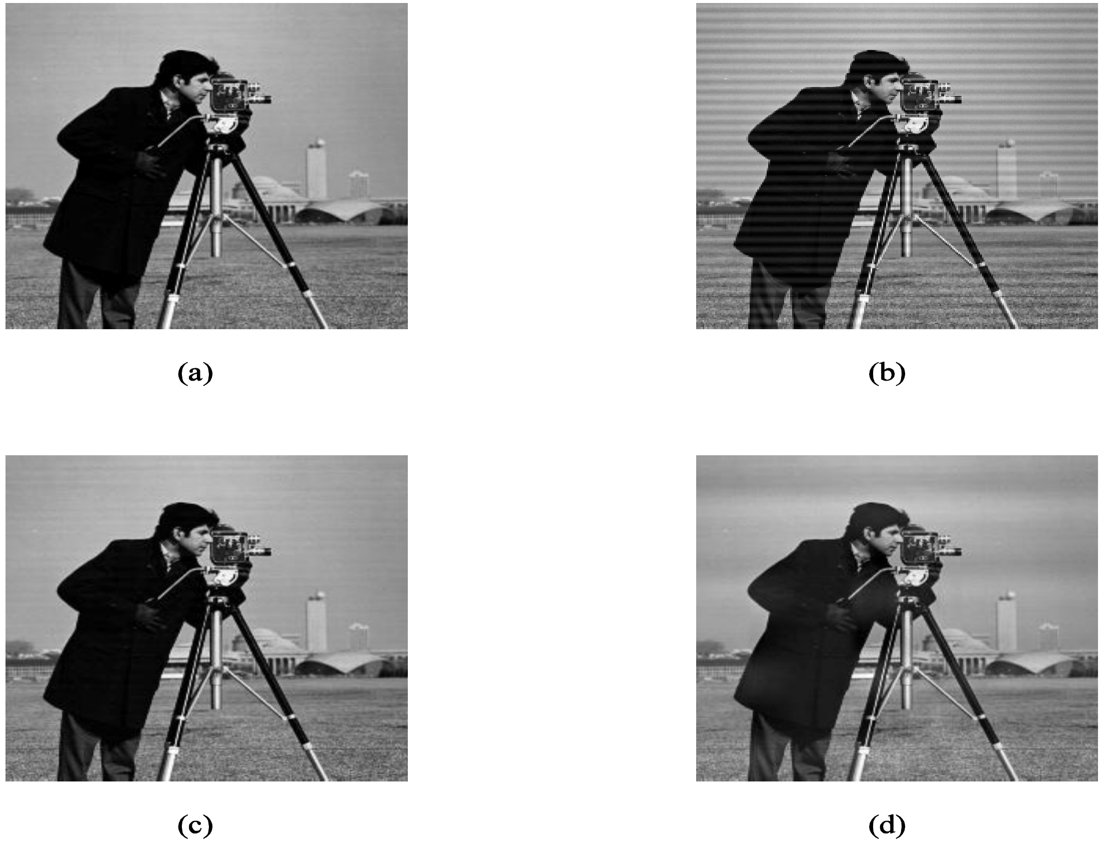


Fig 16. (a) The Cameraman's image, (b) image corrupted with sin(64t) along the rows, (c) filtered image by the sin(64t)-induced DscHT, and (d) filtered image by the sin(8t)-signal DsiHT over the image

FILTRATION OF 2-D IMAGES

(a) (b) (c)

Fig 17. (a) The Cameraman's image (b) image corrupted with a Gaussian noise, and (c) the detected Gaussian noise from the corrupted image with the help of DsiHT (the noise image has been scaled).

Signal with a noise and DsiHT

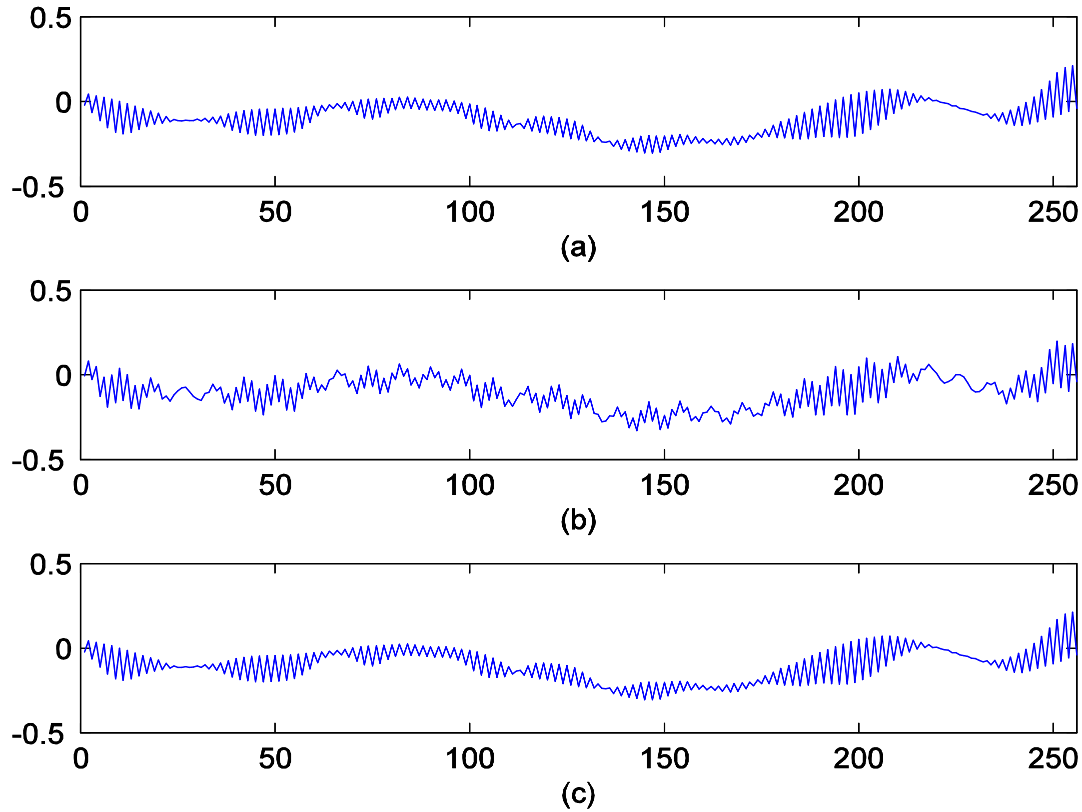


Fig. 18. (a) The 256 samples of the original audio signal, (b) audio signal is mixed with the sin(64t), and (c) the filtered signal with heap transformation.

Summary

- Presented is a class of discrete unitary signal-induced transformations which are defined by systems of moving functions. The movement of the basis functions is accomplished with rotation and the angular representation is defined for the signal-generator.
- The transforms are fast, because of a simple form of decomposition of their matrices, and they can be applied for signals of any length.
- The preliminary experimental results show that the heap transform-based method of filtering can be effectively used for filtering images and noise detecting on images.

DsiHT: *Advantages*

- The transforms are linear and fast, because of a simple form of decomposition of their matrices, and they can be applied for signals of any length, as well as images of any size.
- Matrix of the heap transformation is triangle from the 2nd row, and the 1st rows represent the generators itself.
- DsiHT provide the angular representation of signals and images.

Conclusion: *DsiHT*

- Heap transformations represent a subclass of DsiHTs. More general cases with a few generators and decision equations can be also considered for the DsiHT.
- Complete set of the heap transformation represents variable waves which describe a motion in the space of signals. (These waves are not simple sliding windows as in the wavelet theory)
- The vector-generators and paths of the DsiHT are the keys of the transformation.
- The DsiHT can effectively be used in signal and image processing, image encryption, cryptography, and other areas.

References: *DsiHT*

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Web page - DsiHT

- *This presentation in pdf format will be available in the Dr. Grigoryan web page:*

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