Alpha-Rooting Method of Color Image Enhancement by Discrete Quaternion Fourier Transform

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Outline

- Introduction
- Transform-based image enhancement
- Measures of image enhancement
- Enhancement of images by colors
- 2-D discrete quaternion Fourier transform
- Examples
- Summary

Introduction

- The Fourier transform-based method is one of the image enhancement methods, which in many cases is separately applied to each color plane-component of the color image.
- In this paper, we use the *a*-rooting method of image enhancement in the subspace of quaternion numbers. The concept of the 2-D discrete quaternion Fourier transform (DQFT) is used to perform the *a*-rooting of color images (in RGB format).
- Preliminary results show that the application of the 2-D DQFT plus *a*-rooting method results in high quality color images, and therefore, it can be effectively used for enhancing color images together with the 2-D DFT.

TRANSFORM-BASED IMAGE ENHANCEMENT

The basic idea behind the frequency domain methods consists in computing a discrete unitary transform of the image, for instance the 2-D DFT, manipulating the transform coefficients by the operator M of magnitude, and performing the inverse transform

$$\{f_{n,m}\} \to \{F_{p,s} = |F_{p,s}| e^{-j\vartheta_{p,s}}\} \to \mathrm{M} \circ F \to \{\widehat{F}_{p,s} = M\left[|F_{p,s}|\right] e^{-j\vartheta_{p,s}}\} \to \{\widehat{f}_{n,m}\}.$$

a-rooting method of image enhancement:

$$|F_{p,s}| \to M\left[|F_{p,s}|\right] = |F_{p,s}|^{\alpha}, \quad \alpha \in (0,1)$$

Quantitative Measure of Image Enhancement: General case

To measure the quality of images and select optimal processing parameters, we consider the quantitative measure that relates to Weber's law of human visual system.

The quantitative measure of enhancement of the image processed by Φ transform $\{f_{n,m}\} \to \{\hat{f}_{n,m}\}$

$$EME_{\mathbf{a},\Phi}(\hat{f}) = EME_{L_1,L_2;\mathbf{a},\Phi}(\hat{f}) = \frac{1}{k_1 k_2} \sum_{k=1}^{k_1} \sum_{l=1}^{k_2} 20 \log_{10} \left[\frac{\max_{k,l}(\hat{f})}{\min_{k,l}(\hat{f})} \right]$$

The image $f_{n,m}$ of size $N_1 \times N_2$ is divided by $k_1 k_2$ blocks of size $L_1 \times L_2$.

This measure can be used for selecting the best parameters for image enhancement by the Fourier transform, as well as other unitary transforms

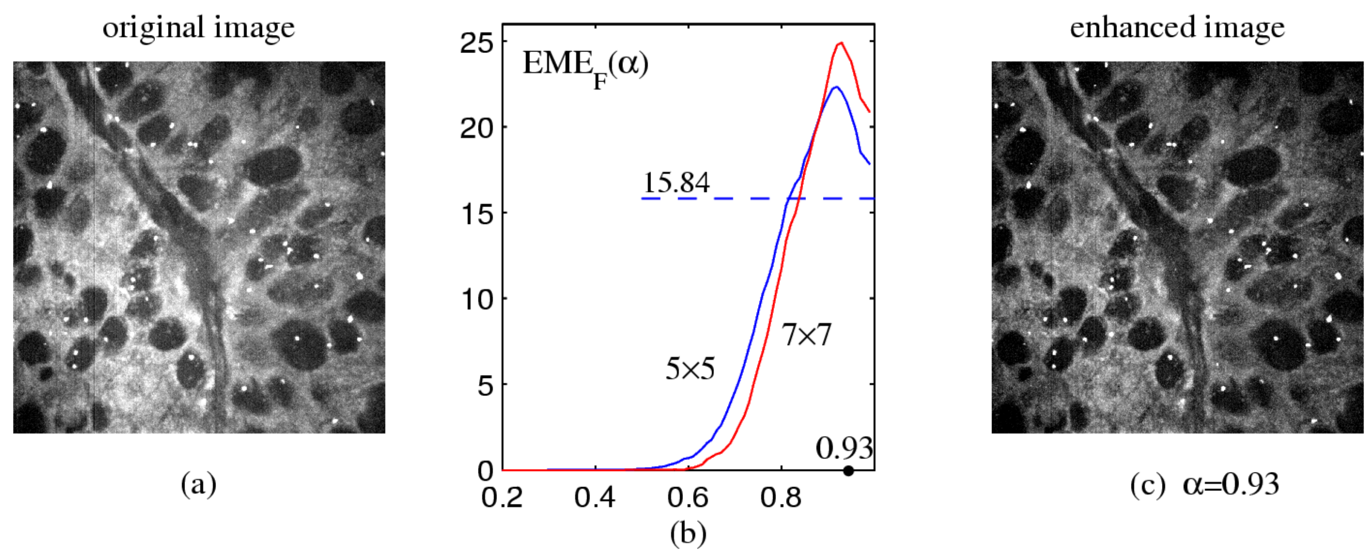


Fig. 1. Parameterized image-enhancement based on the Fourier transform by the *a*-rooting method. (b) Two curves EME(a), (a) FISH image, and (c) image enhanced by the 0.93-rooting method.

The enhancement equals

$$EME_{0.88}(\hat{f}) - EME(f) = 19.61 - 17.11 = 2.50$$

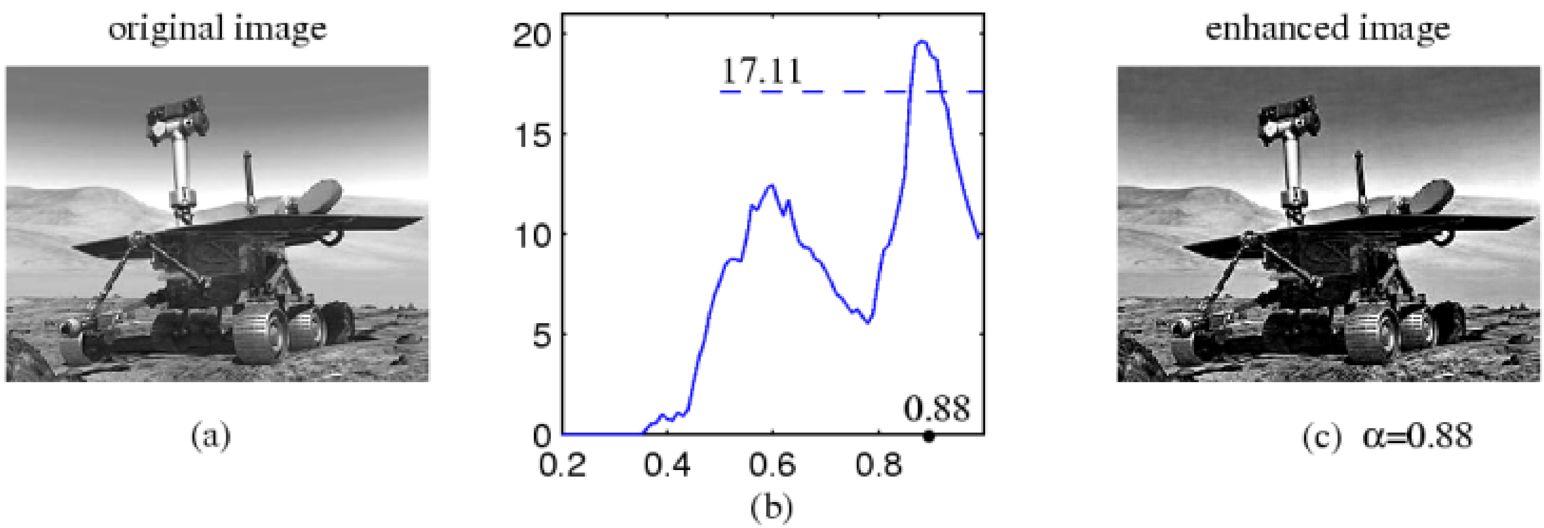


Fig. 2. (a) The original image, (b) the curve EME(α), and (c) image enhanced by the α-rooting method.

Enhancement of Images by colors

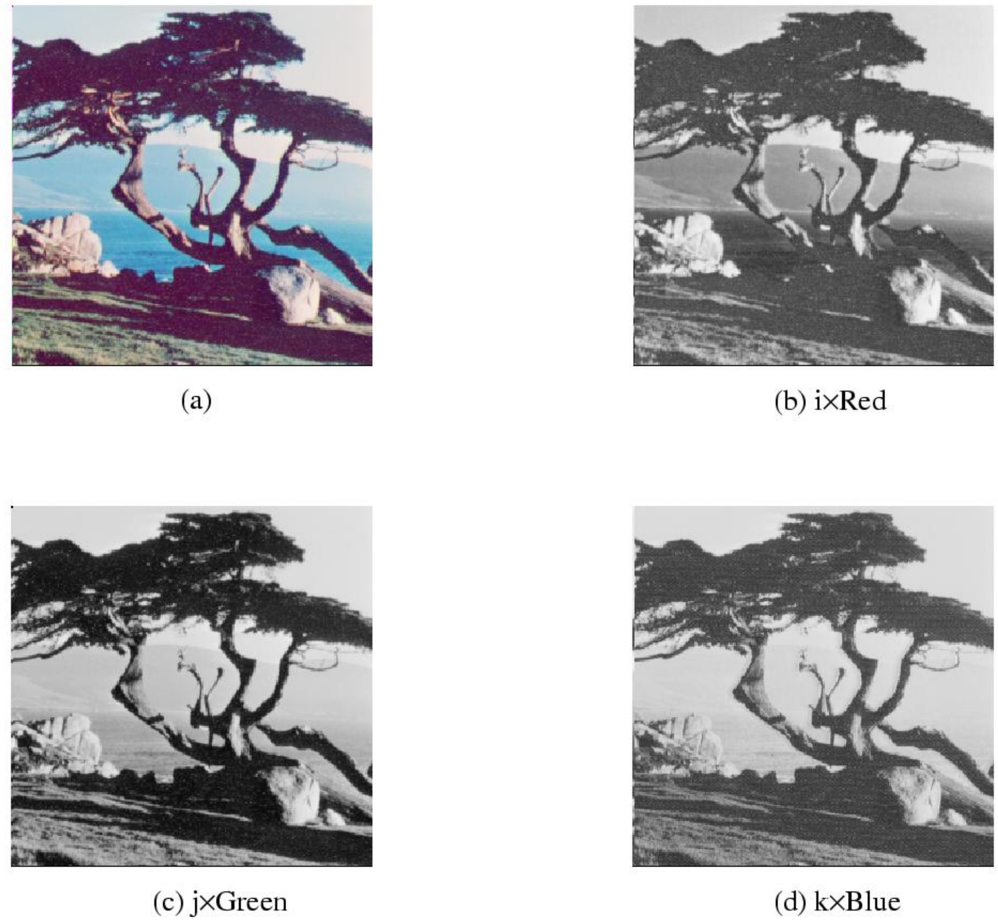


Fig. 3. (a) Color tree image in RGB format, and (b) red, (c) green, and (d) blue channels of the image.

Processing of the Red channel

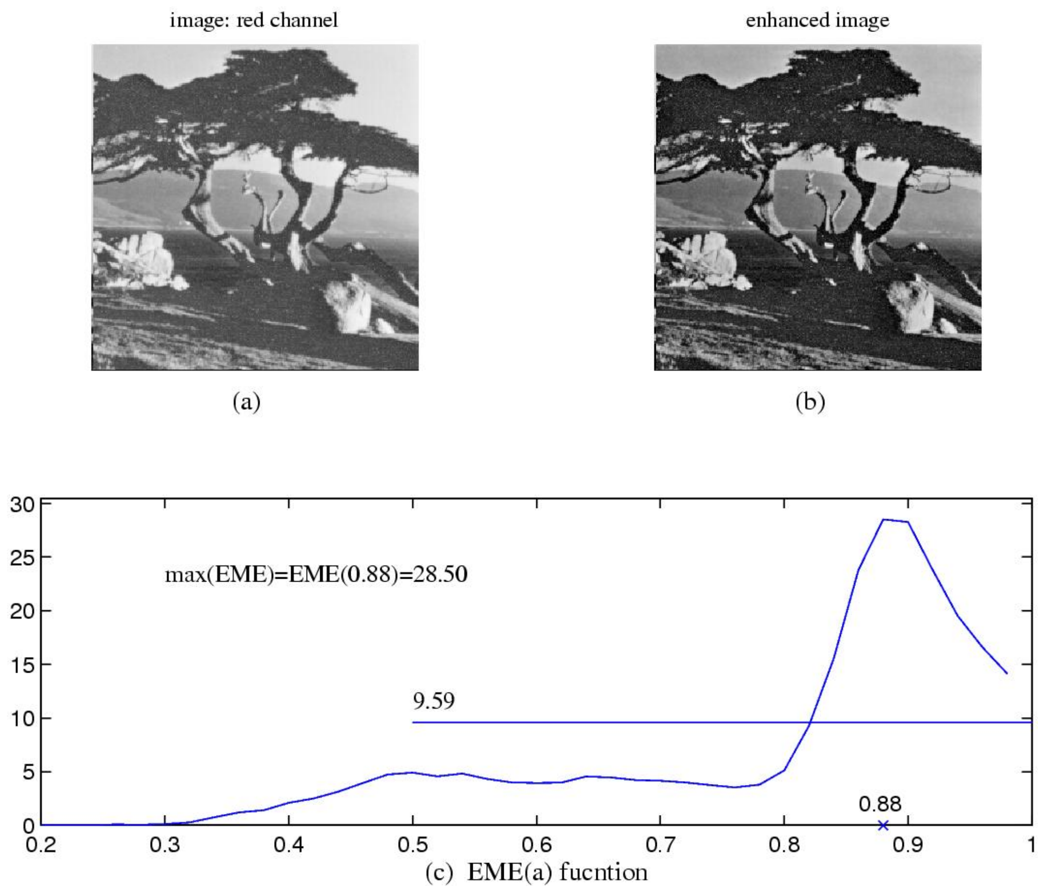


Fig. 4. (a) Red channel of the image, and (b) 0.88-rooting enhancement, and (c) the function EME(a).

Processing of the Green channel

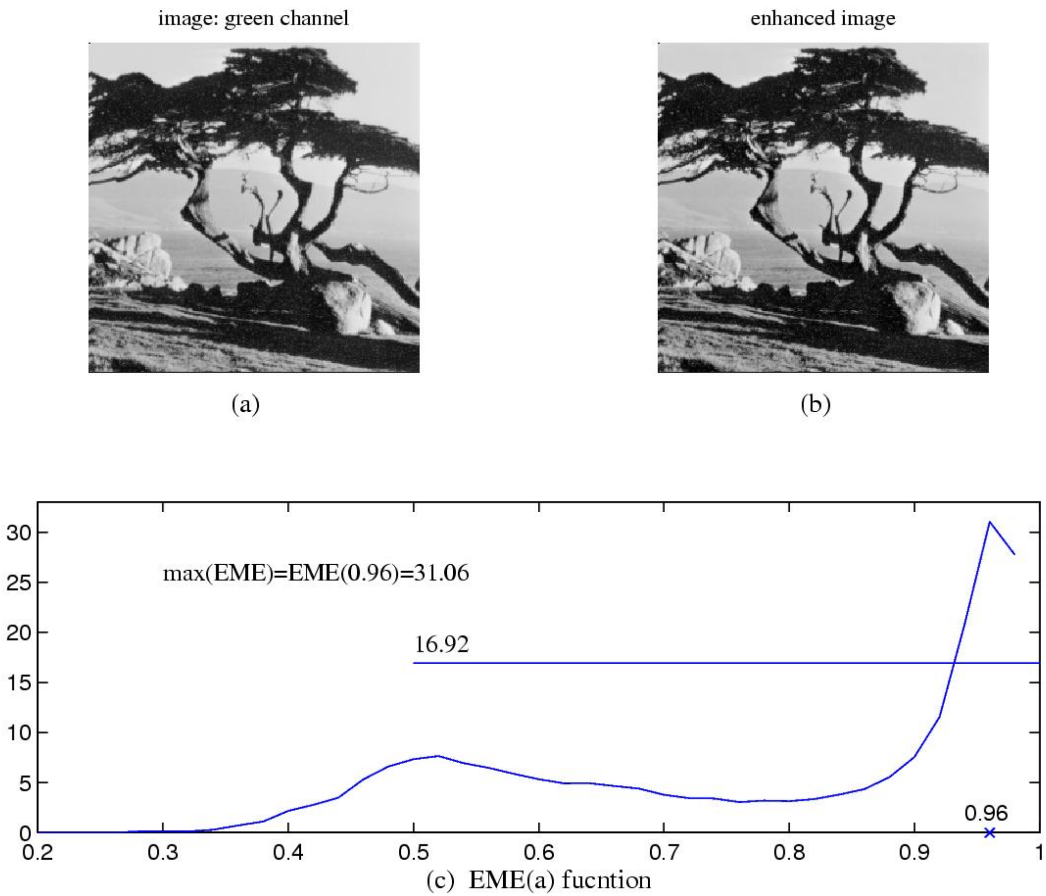


Fig. 5. (a) Green channel of the image, and (b) 0.96-rooting enhancement, and (c) the function EME(a).

Processing of the Blue channel

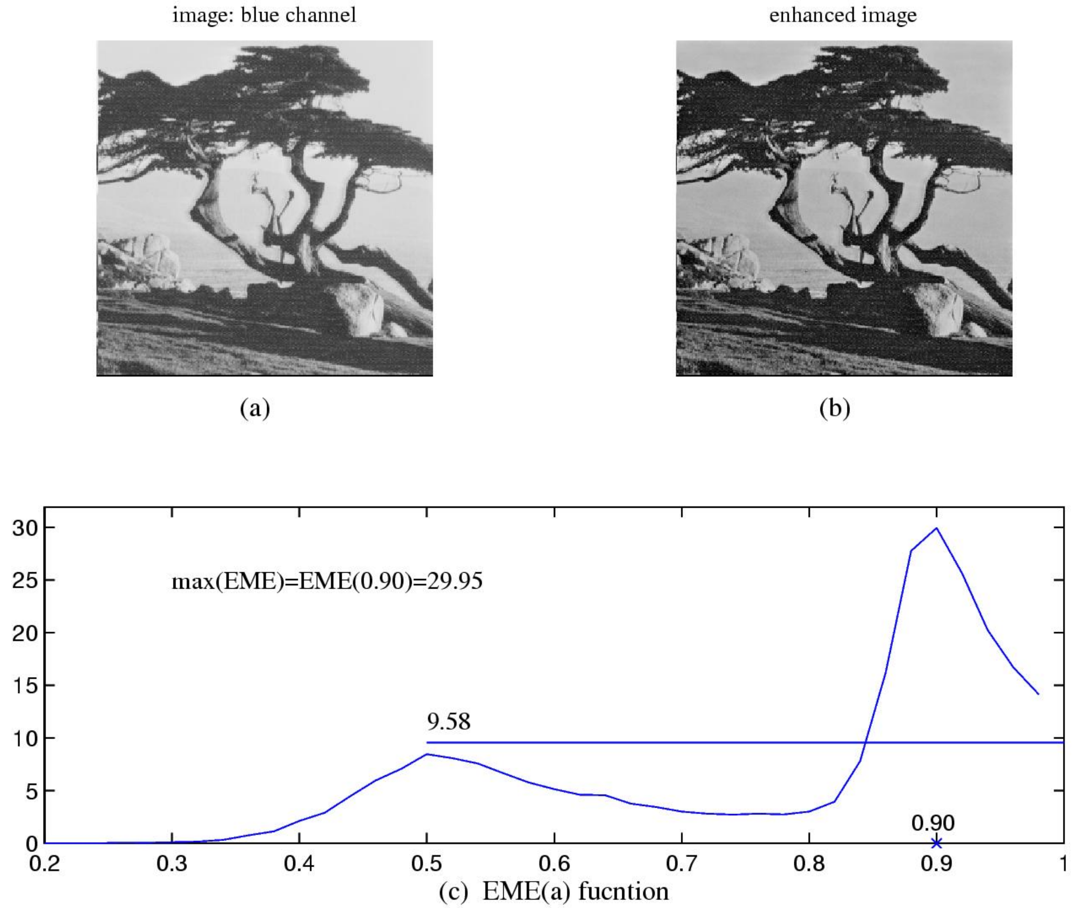


Fig. 6. (a) Blue channel of the image, and (b) 0.90-rooting enhancement, and (c) the function EME(a).

Processing by only one color channel

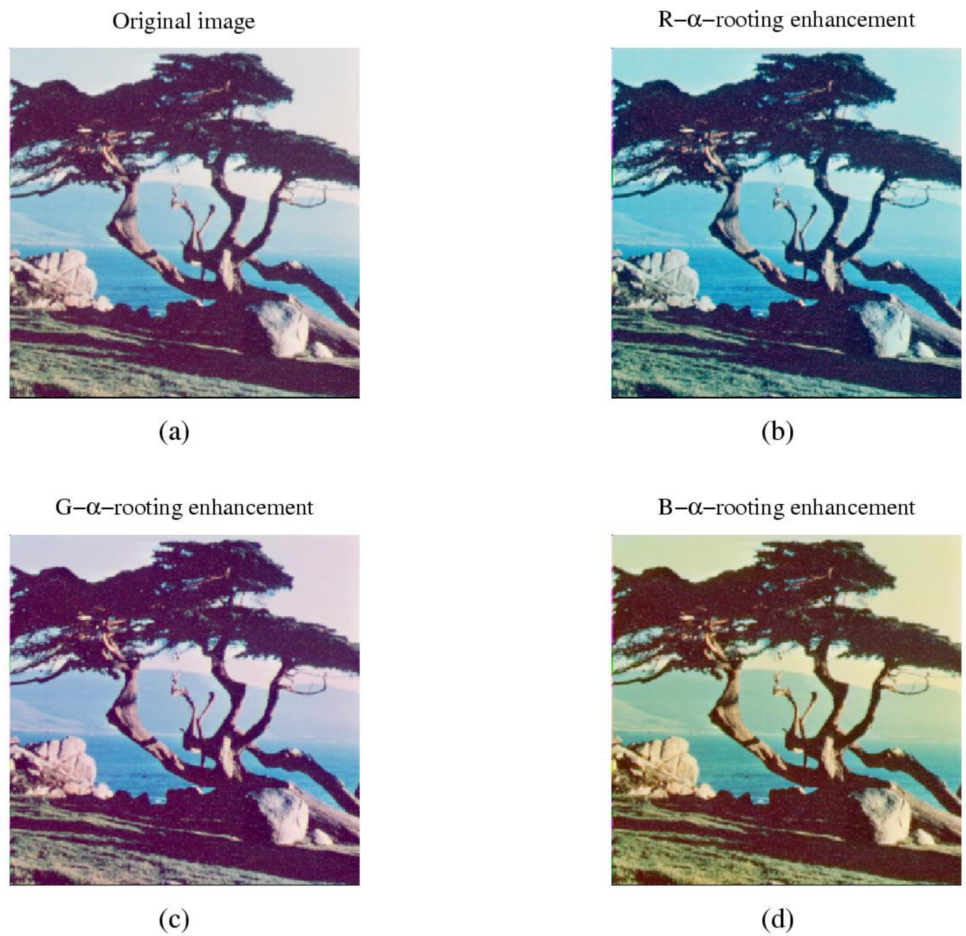


Fig. 7. (a) The image, and the 0.88-, 0.96-, and 0.90-rooting enhancements of the image by only (b) red, (c) green, and (d) blue channels.

Average of the "optimal" processed three channels

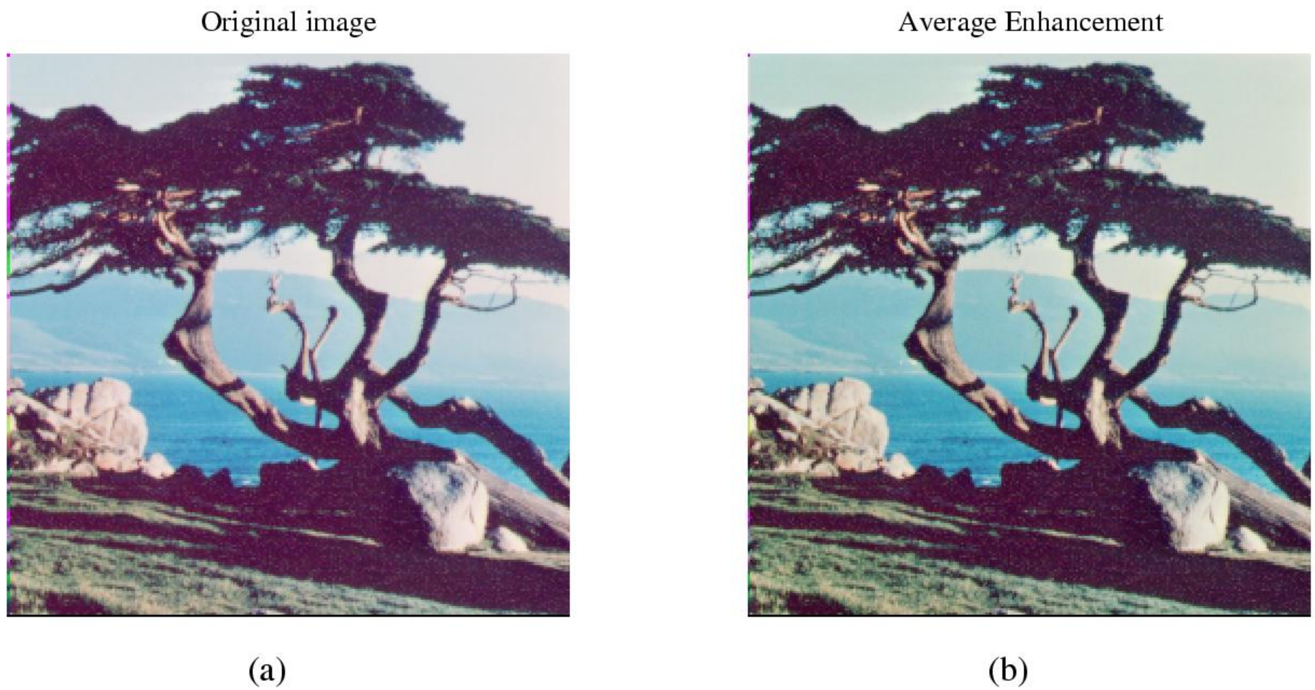


Fig. 8. (a) The tree image and (b) the average of the 0.88-, 0.96-, and 0.90-rooting enhancement of three channels.

The [aR,aG,aB]-rooting enhancement by three channels

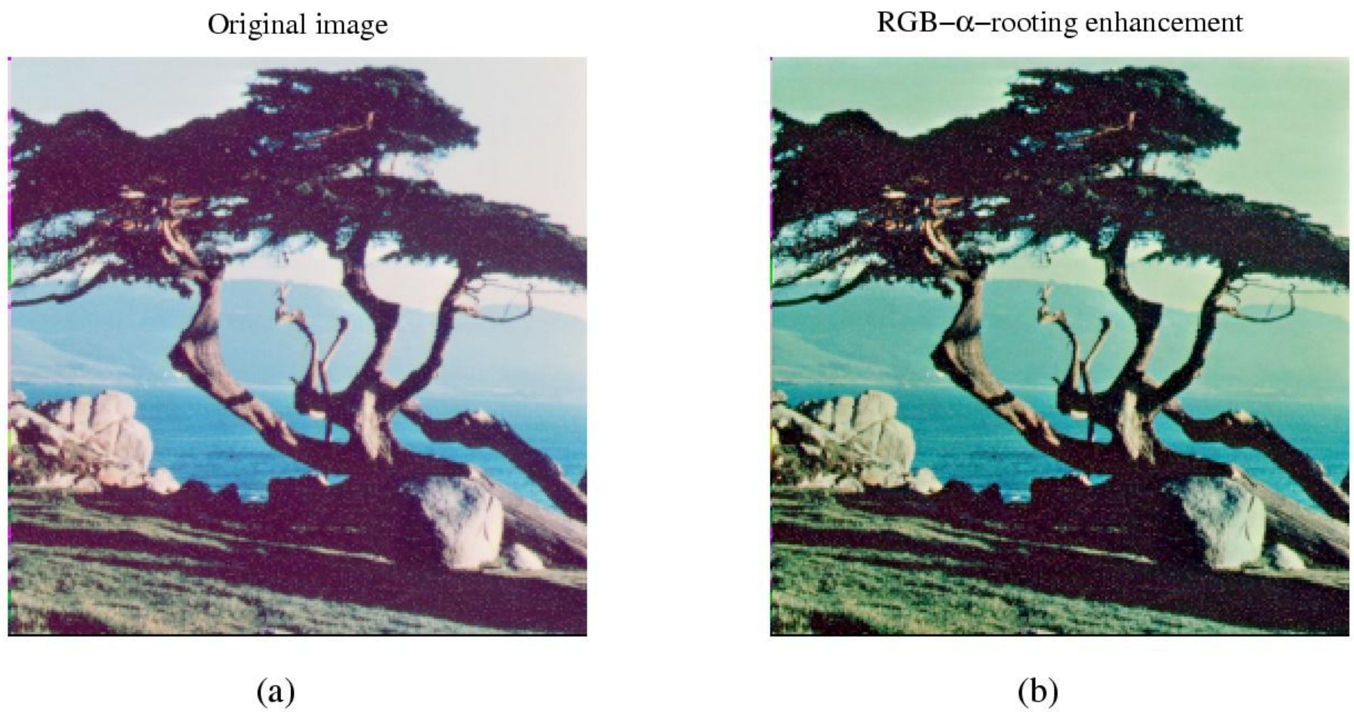


Fig. 9. (a) The tree image and (b) [0.88,0.96,0.90]-rooting enhancement by three channels.

The [aR,aG,aB]-rooting enhancement by three channels

Fig. 10. (a) Original image and (b) [0.82,0.80,0.81]-rooting enhancement by three channels.

Quaternion Numbers

The quaternion can be considered as a 4-D generation of a complex number with one real part and three imaginary parts

$$Q = a + bi + cj + dk = a + (bi + cj + dk),$$

with three imaginary units *i*, *j*, and *k*.

$$ij = -ji = k, \quad jk = -kj = i,$$
$$ki = -ik = -j, \quad i^2 = j^2 = k^2 = ijk = -1.$$

Quaternion conjugate and modulus are

$$\bar{Q} = a - (bi + cj + dk), \quad |Q| = \sqrt{a^2 + b^2 + c^2 + d^2}.$$

Color image and Quaternion numbers

The discrete color image in the RGB color space can be transformed into imaginary part of quaternion numbers by encoding the components of the RGB value as:

$$f(n,m) = R(n,m)i + G(n,m)j + B(n,m)k.$$

The advantage of using quaternion based operations is that we do not have to process each color channel independently, but rather, <u>treat each color triple as a whole unit</u>.

Two-side 2-D Quaternion Fourier Transform

$$F_{p,s} = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} W_j^{np} f_{n,m} W_k^{ms} = \sum_{n=0}^{N-1} W_j^{np}\left(\sum_{m=0}^{M-1} f_{n,m} W_k^{ms}\right)$$

$$p = 0:(N-1),\ s = 0:(M-1)$$

$$W_j = \cos(2\pi/N) - j\sin(2\pi/N), \quad W_k = \cos(2\pi/M) - k\sin(2\pi/M)$$

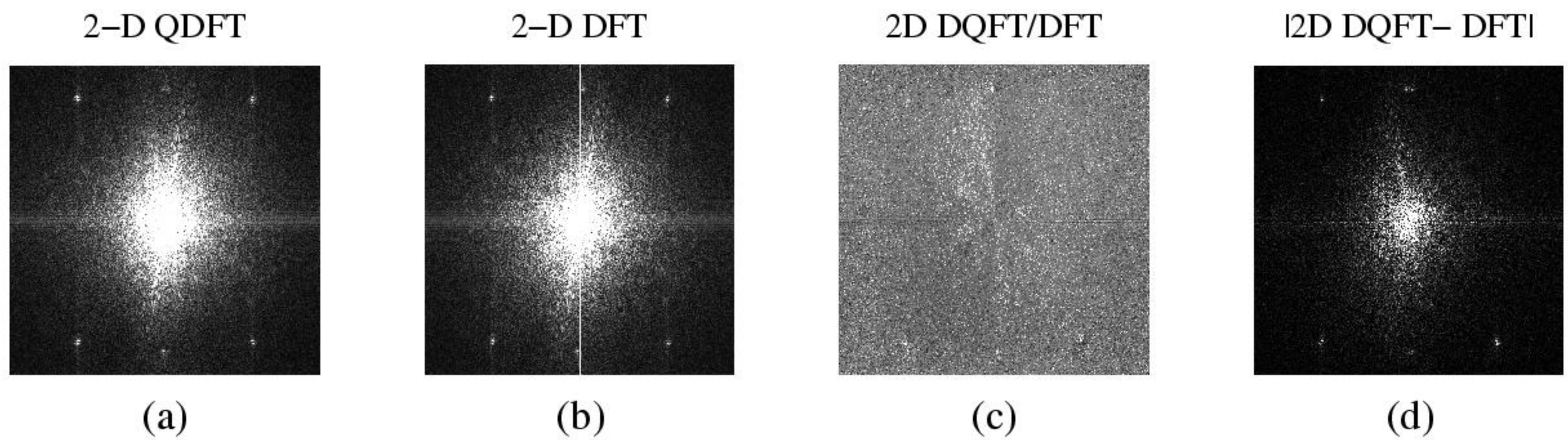


Fig. 12. Magnitudes of (a) the 2-D QDFT and (b) the 2-D DFT of the color tree image, (c) the ratio and (d) difference of magnitudes.

2-D DQFT: multiplication

Each multiplication in the 2-D DQFT

$$\begin{aligned} Q &= W_j^{np} f_{n,m} W_k^{ms} \\ &= Q_r + iQ_i + jQ_j + kQ_k \\ &= [\cos(\varphi) - j\sin(\varphi)][a + ib + jc + kd][\cos(\psi) - k\sin(\psi)] \\ &\quad (\ \varphi = 2\pi np/N, \quad \psi = 2\pi ms/M\ ) \end{aligned}$$

can be defined in matrix form as

$$Q = \begin{pmatrix} Q_r \\ Q_i \\ Q_j \\ Q_k \end{pmatrix} = \begin{pmatrix} \cos(\varphi)\cos(\psi) & -\sin(\varphi)\sin(\psi) & \sin(\varphi)\cos(\psi) & \cos(\varphi)\sin(\psi) \\ \sin(\varphi)\sin(\psi) & \cos(\varphi)\cos(\psi) & -\cos(\varphi)\sin(\psi) & -\sin(\varphi)\cos(\psi) \\ -\sin(\varphi)\cos(\psi) & \cos(\varphi)\sin(\psi) & \cos(\varphi)\cos(\psi) & -\sin(\varphi)\sin(\psi) \\ -\cos(\varphi)\sin(\psi) & \sin(\varphi)\cos(\psi) & \sin(\varphi)\sin(\psi) & \cos(\varphi)\cos(\psi) \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

where $a+ib+jc+kd$ stands for $f_{n,m}$.

Quaternion Transform-Based α-rooting Image Enhancement Algorithm

Input is a color image and $\alpha \epsilon [0,1]$.

- **Step 1**. Perform the 2-D DQFT of the color image.
- **Step 2**. Multiply the transform coefficients, $F_{p,s}$, by the quaternion factors $C(p,s)=c|F_{p,s}|^{\alpha-1}$.
- **Step 3**. Perform the inverse 2-D DQFT.

Output is an enhanced color image.

Image enhancement: Examples

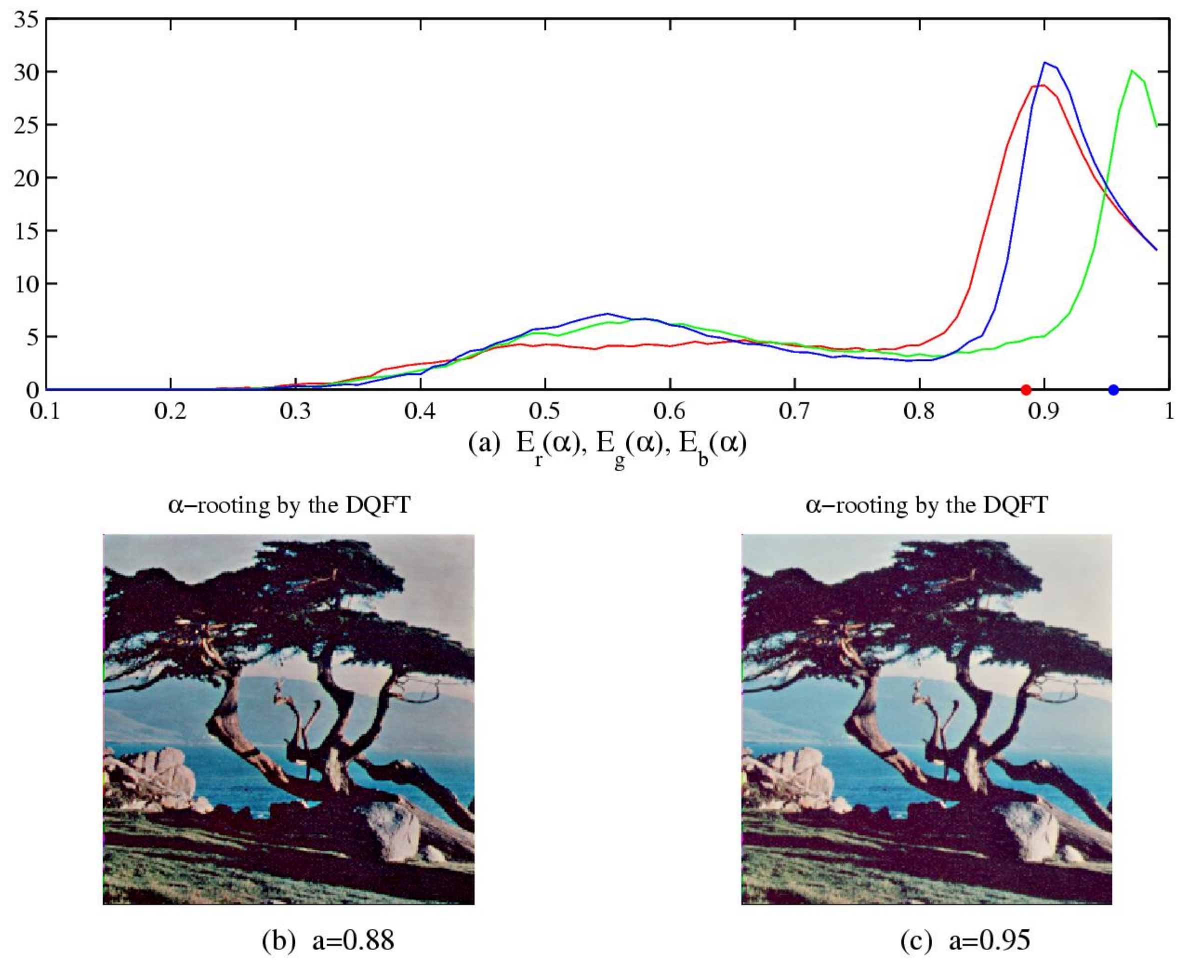


Fig. 13. (a) Enhancement functions for three channels of the tree image. (b,c) The α-rooting by the 2-D DQFT.

Image enhancement: Examples

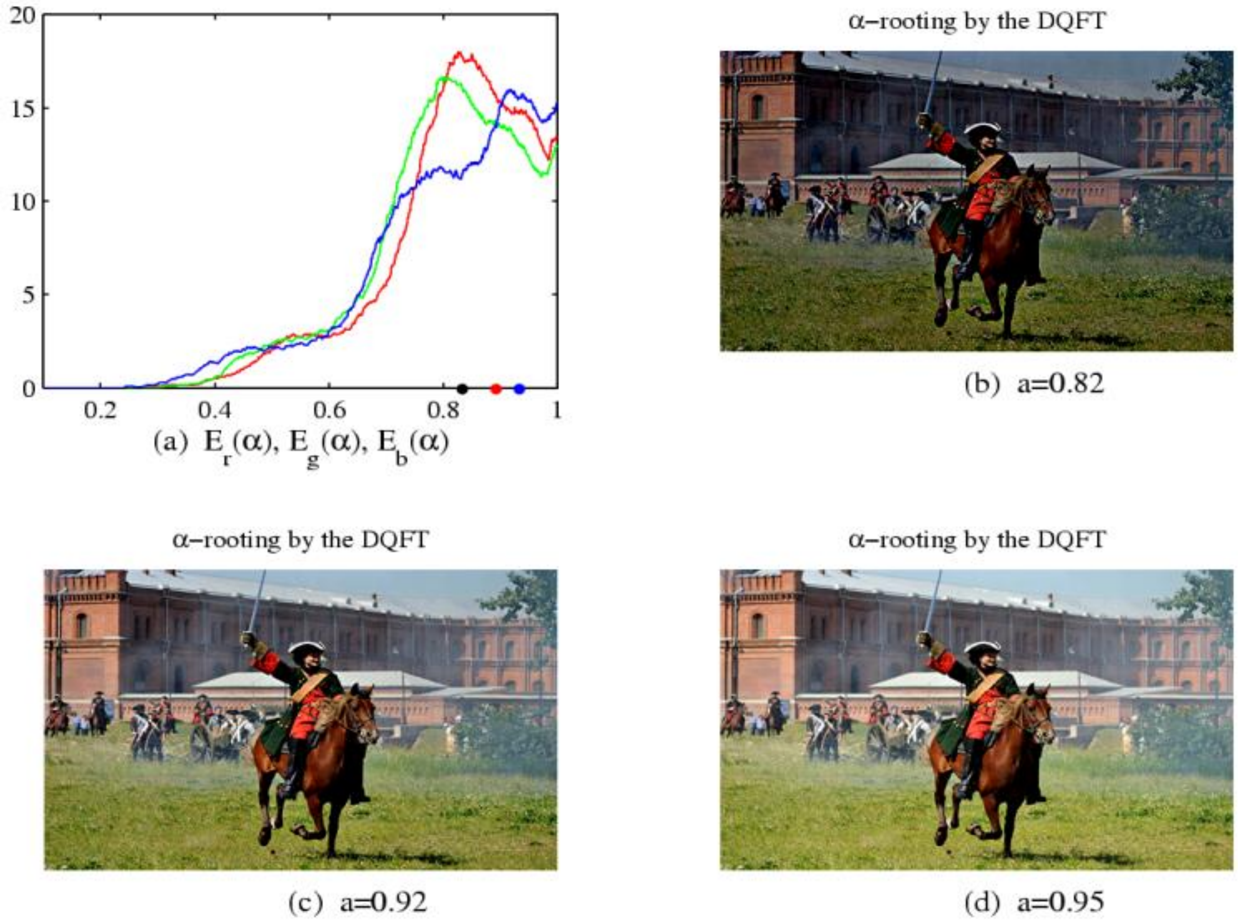


Fig. 14. (a) Enhancement functions for three channels of the tree image. (b,c) The α-rooting by the 2-D DQFT.

Image enhancement: Examples

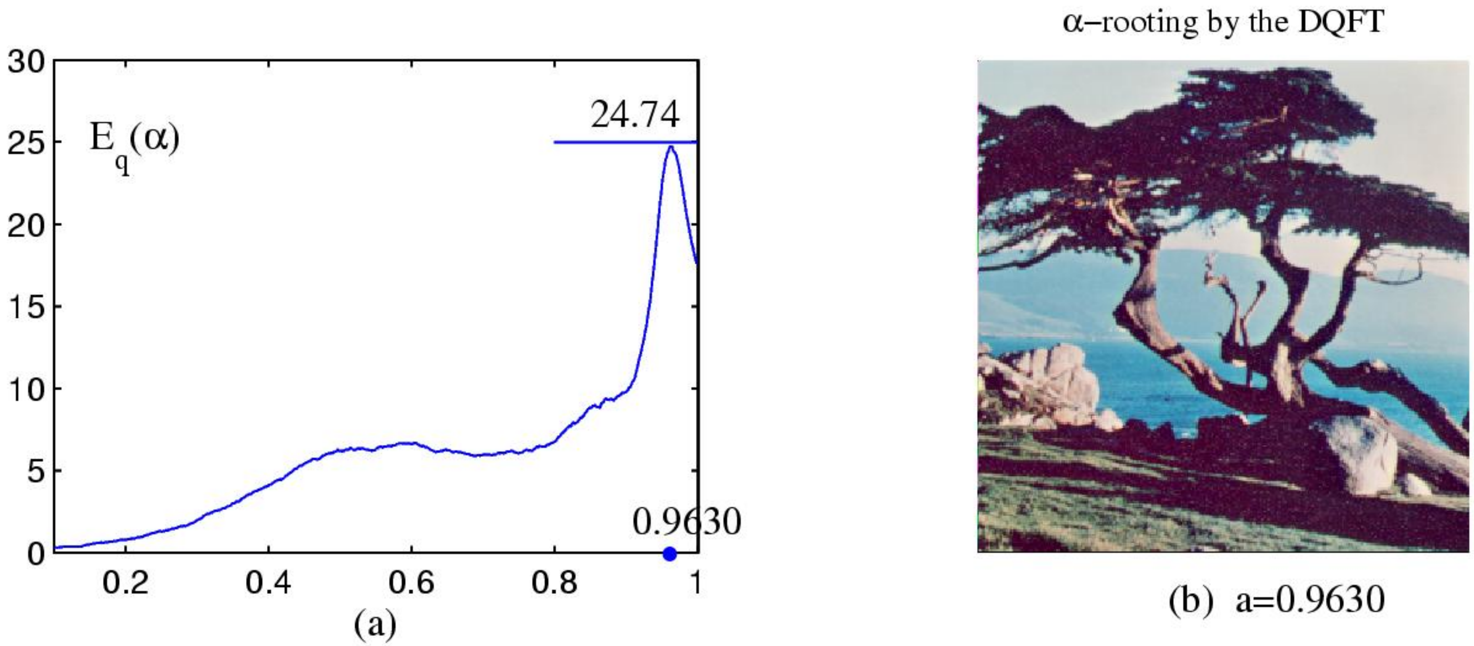


Fig. 15. (a) Enhancement function of the tree image.
(b) The 0.9630-rooting by the 2-D DQFT.

Image enhancement: Examples

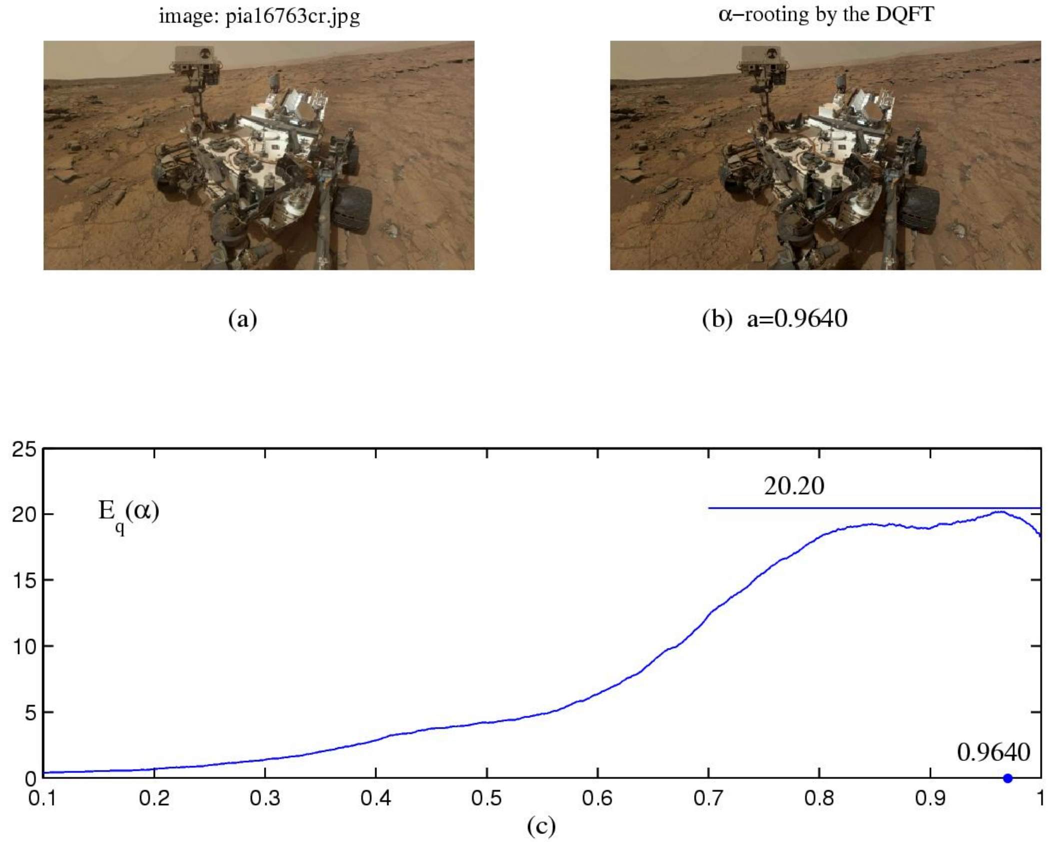


Fig. 16. (a) 466×248×3 roller image, (b) 0.964-rooting by the 2-D DQFT, (c) the graph of $E_q(\alpha)$.

Image enhancement: Examples

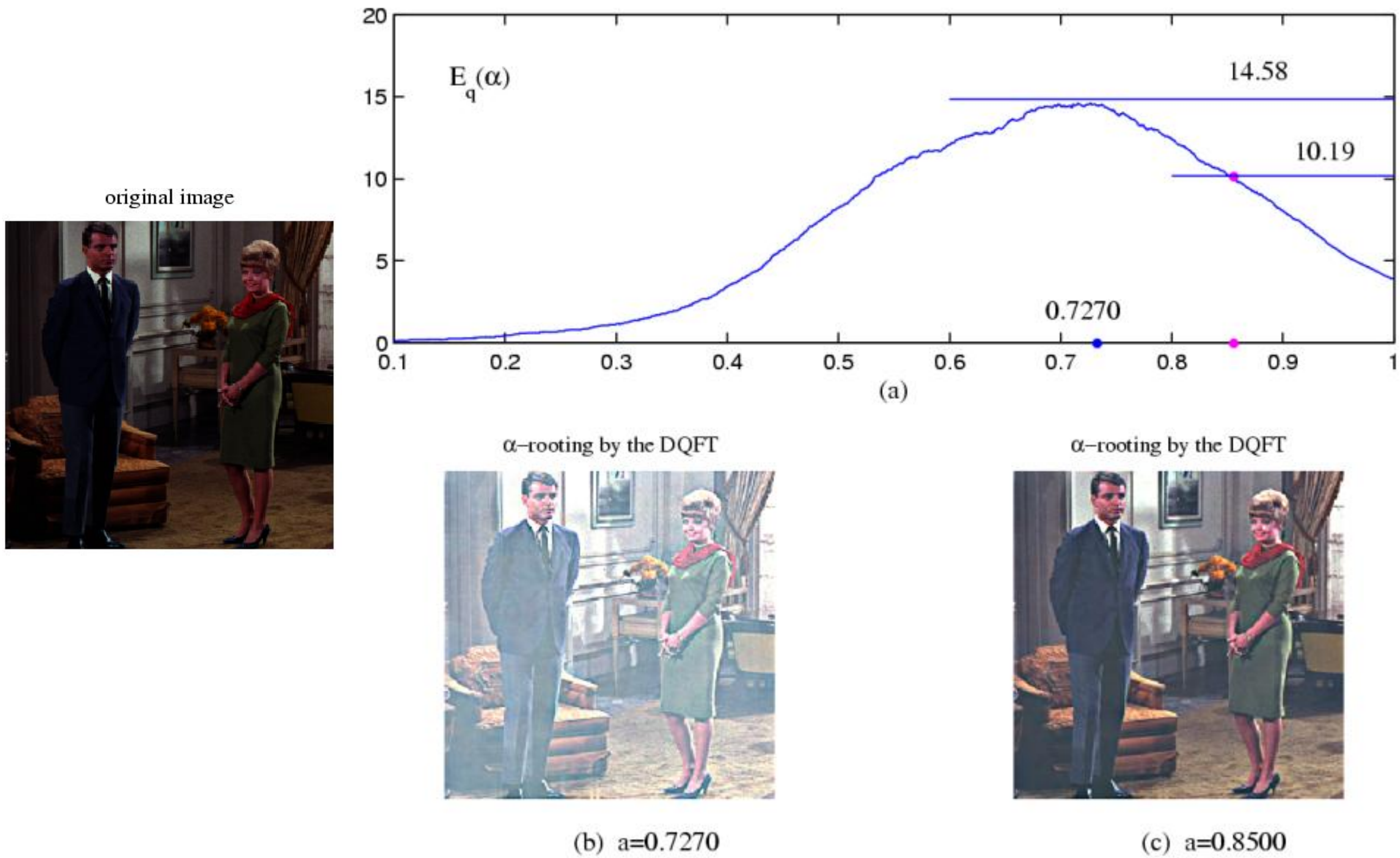


Fig. 17. (a) Enhancement functions for three channels. The α-rooting by the 2-D DQFT for (b) α=0.7270 and (c) α=0.850.

Image enhancement: Examples

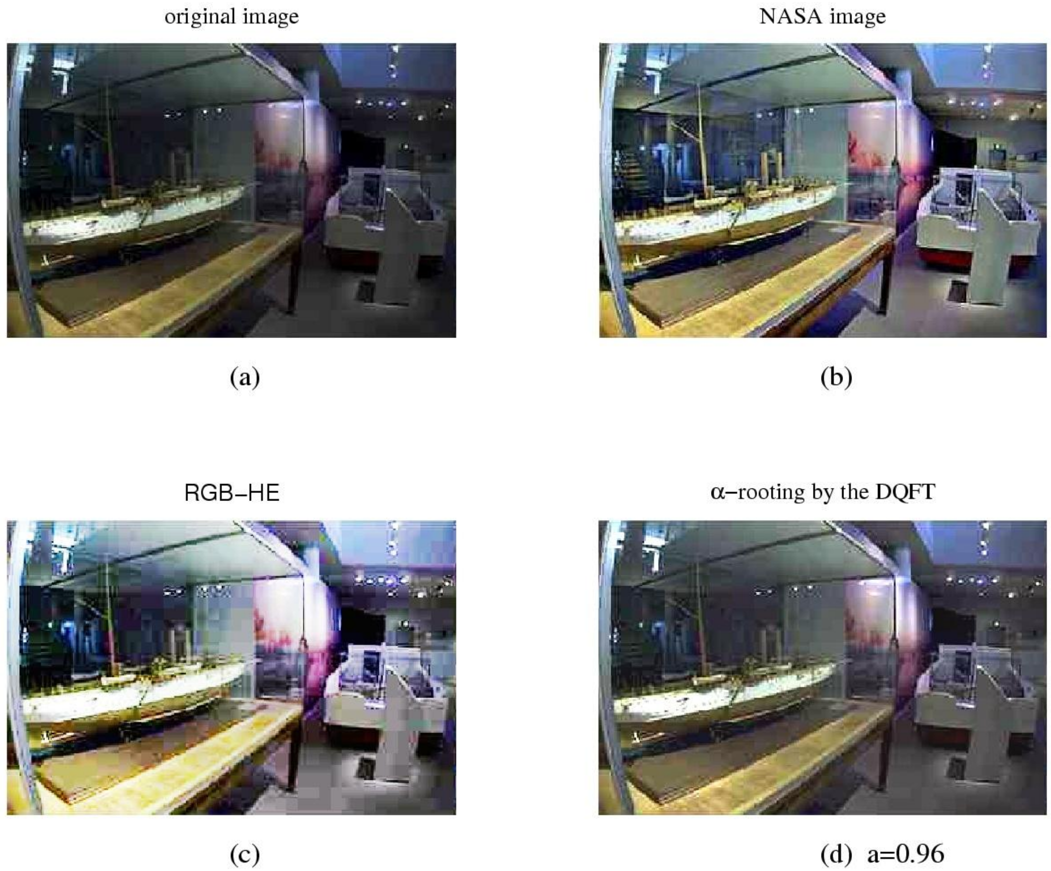
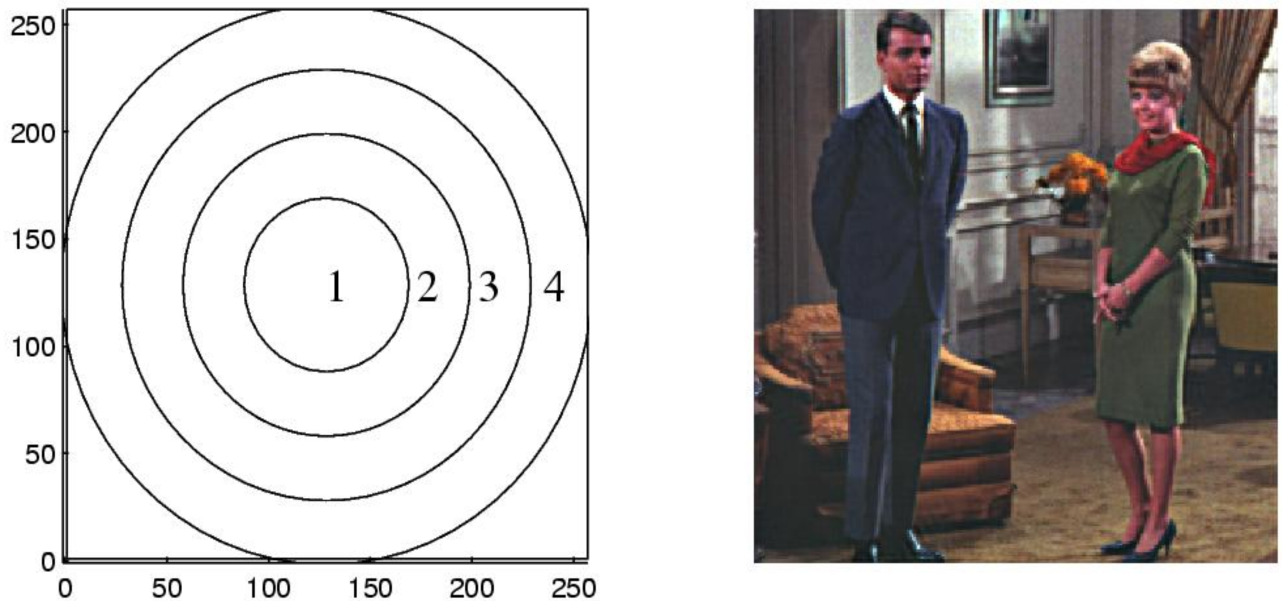


Fig. 18. (a) The room image, (b) Retinex enhancement, (c) 3 channel HE, (d) 0.96-rooting by the 2-D DQFT.

Summary

- Presented is a new methods for color image enhancement based on the 2-D DQFT. The new methods are well-suited for color image processing applications; it processes all three color components (R,G,B) simultaneously.
- Proposed is the so-called CEME measure to evaluate the quality of color enhanced images.
- Shown is that this method has a better performance of color image enhancement in comparison with other recent methods. The proposed algorithms are simple to apply and design, which makes them practical.

Multi-frequency band α-rooting



(a) (b)

- Fig. X. (a) 2-D frequency lattice divided by five bands and (b) band α-rooting by the 2-D DQFT.
- α_1=0.90, α_2=0.87, α_3=0.85, α_4=0.82, α_5=0.90.