Alpha-Rooting Method of Gray-scale Image Enhancement in The Quaternion Frequency Domain

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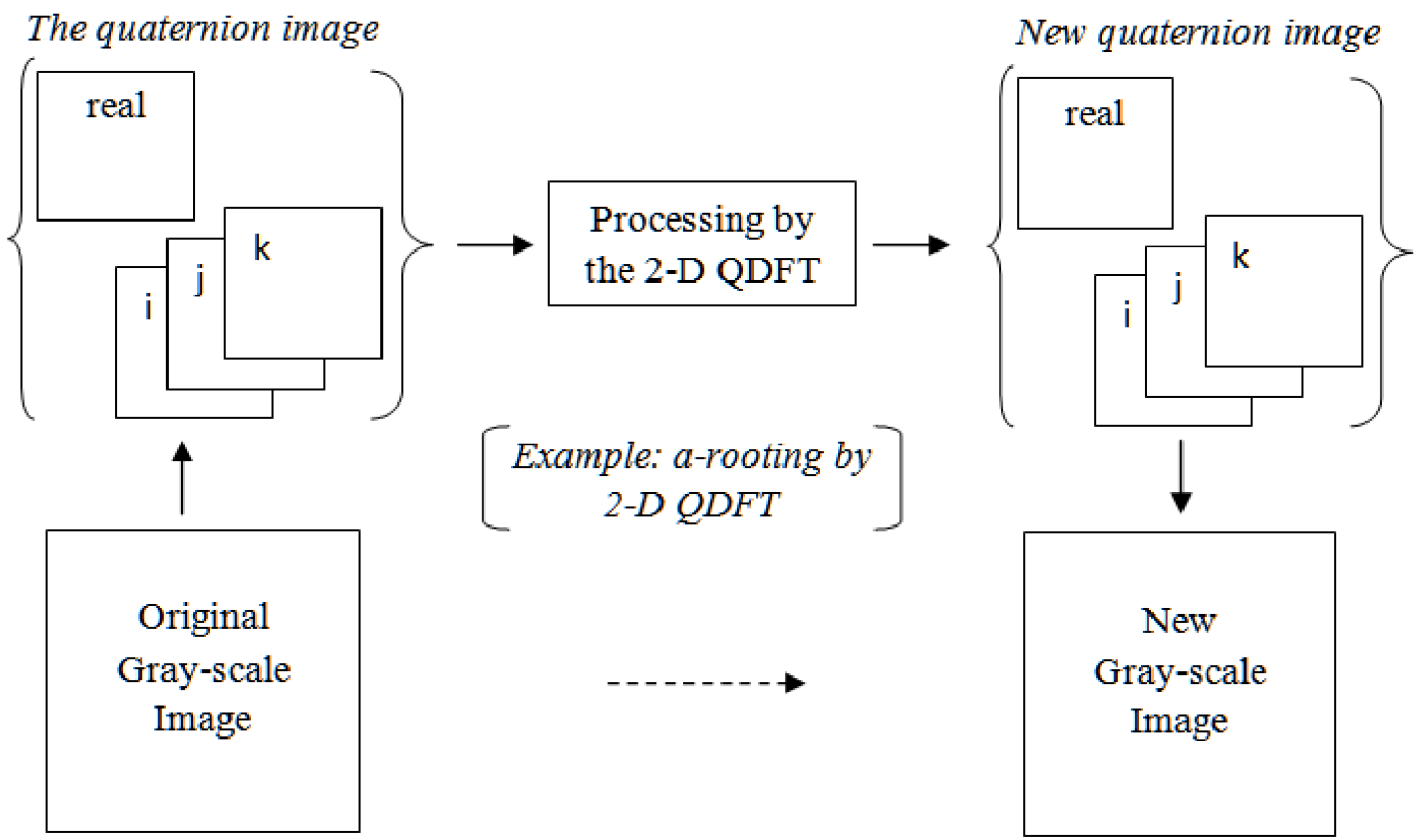
Introduction

- New gray-scale image representations in the 4-D quaternion space allows for images to be processed as quaternion and color images in the frequency domain, by using the concepts of the 2-D quaternion unitary transforms, such as quaternion discrete Fourier transforms, wavelet, and others.
- In this paper, we use the *a*-rooting method of image enhancement by the 2-D discrete quaternion Fourier transform (DQFT).
- Preliminary results show: that the application of new models of gray-scale images in the quaternion space results in high quality gray-scale images, and can be effectively used for enhancing images.

Block-diagram of Gray-scale Imaging



- **Fig. 1** The block-diagram of gray-scale image processing in the frequency domain.

Diagram: Main Steps

1. Gray scale image is transformed into the quaternion image of size smaller than the original image,

$$\{f_{n,m};\, n=0:(N-1),\ m=0:(M-1)\} \rightarrow \{q_{n,m};\, n=0:(N_1-1),\ m=0:(M_1-1)\},$$

where

$$q_{n,m} = a_{n,m} + (i(q_i)_{n,m} + j(q_j)_{n,m} + k(q_k)_{n,m}) \quad i^2=j^2=k^2=-1.$$

2. The quaternion image is processed in the frequency domain

$$q_{n,m} \rightarrow \hat{q}_{n,m}, \quad n=0:(N_1-1),\ m=0:(M_1-1)$$

by using for instance, the concept of the 2-D QDFT.

3. The processed quaternion image is transformed back into gray-scales

$$\{\hat{q}_{n,m};\, n=0:(N_1-1),\ m=0:(M_1-1)\} \rightarrow \{\hat{f}_{n,m};\, n=0:(N-1),\ m=0:(M-1)\}$$

Quaternion Numbers

The quaternion can be considered as a 4-D generation of a complex number with one real part and three imaginary parts

$$Q = a + bi + cj + dk = a + (bi + cj + dk),$$

with three imaginary units *i*, *j*, and *k*.

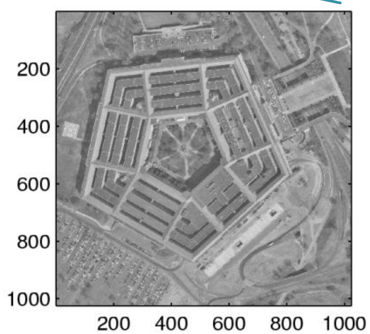
$$ij = -ji = k, \quad jk = -kj = i,$$
$$ki = -ik = -j, \quad i^2 = j^2 = k^2 = ijk = -1.$$

Quaternion conjugate and modulus are

$$\bar{Q} = a - (bi + cj + dk), \quad |Q| = \sqrt{a^2 + b^2 + c^2 + d^2}.$$

Gray-Scale-To-Quaternion Image Model

Model : Image $f_{n,m}$ of size $N\times M$ can be divided by 4 parts



.	.	.	.	.	.
.	$f_{2n,2m}$	$f_{2n,2m+1}$	$f_{2n,2m+2}$	$f_{2n,2m+3}$	.
.	$f_{2n+1,2m}$	$f_{2n+1,2m+1}$	$f_{2n+1,2m+2}$	$f_{2n+1,2m+3}$	.
.	$f_{2n+2,2m}$	$f_{2n+2,2m+1}$	$f_{2n+2,2m+2}$	$f_{2n+2,2m+3}$	.
.	$f_{2n+3,2m}$	$f_{2n+3,2m+1}$	$f_{2n+3,2m+2}$	$f_{2n+3,2m+3}$	.
.	.	.	.	.	.

$$f_e = \{(f_e)_{n,m}\} = \{f_{2n,2m}\}, \qquad f_i = \{(f_i)_{n,m}\} = \{f_{2n,2m+1}\},$$
$$f_j = \{(f_j)_{n,m}\} = \{f_{2n+1,2m}\}, \qquad f_k = \{(f_k)_{n,m}\} = \{f_{2n+1,2m+1}\},$$

where $n = 0 : (N/2-1)$, $m = 0 : (M/2-1)$.

These four parts can be considered as components of the quaternion matrix which we call the quaternion image and denote by *q(f)*,

$$q(f) = (f_e, f_i, f_j, f_k) = f_e + (if_i + jf_j + kf_k)$$

Gray-Scale-To-Quaternion Image Model

When transforming the gray-scale image into the quarter-nion subspace, we may assume that the components f_i, f_j, and f_k are the set-components of the imaginary part of the quaternion image, as shown in this table

.	.	.	.	.	.
.	**E**: $f_{2n,2m}$	**R**: $f_{2n,2m+1}$	**E**: $_{2n,2m+2}$	**R**: $f_{2n,2m+3}$	.
.	**G**: $f_{2n+1,2m}$	**B**: $f_{2n+1,2m+1}$	**G**: $f_{2n+1,2m+2}$	**B**: $f_{2n+1,2m+3}$	.
.	**E**: $f_{2n+2,2m}$	**R**: $f_{2n+2,2m+1}$	**E**: $f_{2n+2,2m+2}$	**R**: $f_{2n+2,2m+3}$	.
.	**G**: $f_{2n+3,2m}$	**B**: $f_{2n+3,2m+1}$	**G**: $f_{2n+3,2m+2}$	**B**: $f_{2n+3,2m+3}$	.
.	.	.	.	.	.

Table 1. The gray-scale image mapping to the RGB color model:

The letter **E** stands for the gray-levels, **R** for the red color, **G** for the green color, and **B** for the blue color.

We assume that the component f_i is set to the red color channel, and f_j and f_k to the green and blue channels, respectively, as shown in the table.

Gray-Scale-To-Quaternion Image Model: Example

The gray-scale "pentagon" image of size 1024×1024.

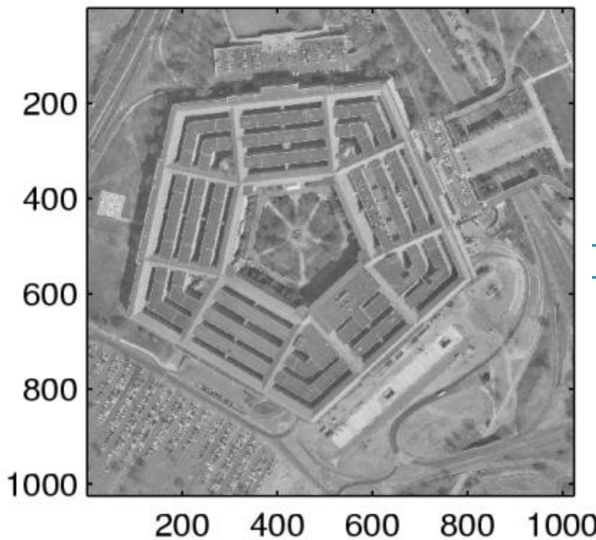
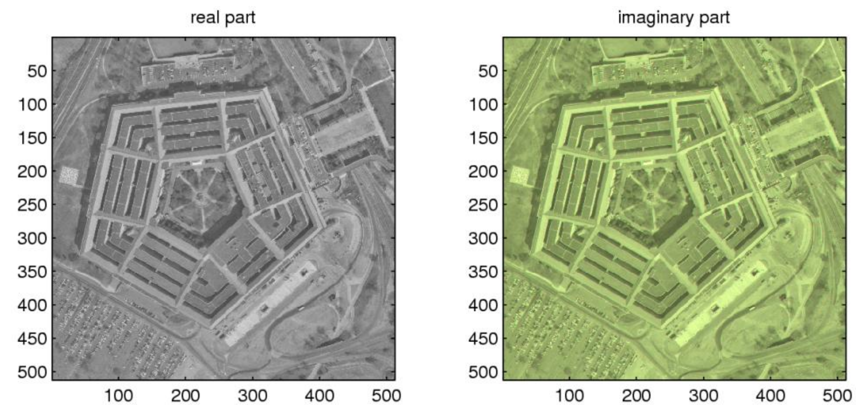


Fig. 2

RGB color model



(a) the real part and (b) the imaginary part

Fig. 3 The quaternion image of size 512×512:

The gray-scale image and color image of size 512×512 each represent the original gray-scale "pentagon" image of size 1024×1024 in the quaternion space.

Quaternion-TRANSFORM-BASED IMAGE ENHANCEMENT

The basic idea behind the frequency domain methods consists in computing a discrete unitary transform of the image, for instance the 2-D quaternion DFT (2-D QDFT), manipulating the transform coefficients by a operator ***M*** of magnitude, and performing the inverse transform.

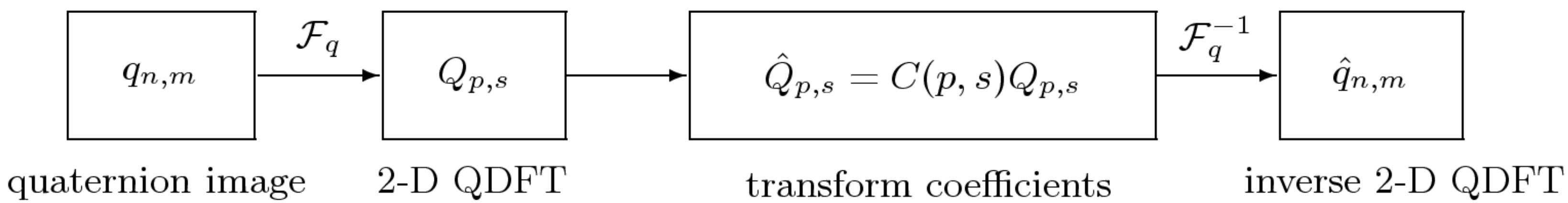


Fig. 4 Block-diagram of the quaternion Fourier transform-based image enhancement

The <u>*a*-rooting</u> method of image enhancement:

$$|Q_(p,s)| \to M(|Q_(p,s)|) = |Q_(p,s)|^{\alpha}, \quad p,s = 0:(N-1).$$

Quaternion Fourier Transform-Based α-Rooting Image Enhancement Algorithm

Input is a quaternion image or color image and the value of the parameter α is from the interval (0,1].

- **Step 1**. Perform the 2-D QDFT of the color image.
- **Step 2**. Multiply the transform coefficients, $Q_{p,s}$, by the quaternion factors $C(p,s)=c|Q_{p,s}|^{\alpha-1}$.
- **Step 3**. Perform the inverse 2-D QDFT.

The output is an enhanced quaternion image.

The imaginary part of the output is an enhanced color image.

Quantitative Measure of Image Enhancement: Gray-scale Image case

To measure the quality of gray-scale images and select "optimal" processing parameter (or parameters) for image enhancement, we consider the quantitative measure, which is known as EME – enhancement mesure estimation.

The quantitative measure of enhancement of the image processed by Φ transform $\{f_{n,m}\} \rightarrow \{\hat{f}_{n,m}\}$

$$EME_{\mathbf{a},\Phi}(\hat{f}) = EME_{L_1,L_2;\mathbf{a},\Phi}(\hat{f}) = \frac{1}{k_1 k_2} \sum_{k=1}^{k_1} \sum_{l=1}^{k_2} 20 \log_{10} \left[\frac{\max_{k,l}(\hat{f})}{\min_{k,l}(\hat{f})} \right]$$

The image $f_{n,m}$ of size $N_1 \times N_2$ is divided by $k_1 k_2$ blocks of size $L_1 \times L_2$, for instance 7×7 or 9×9.

Alpha-rooting – 2-D DFT Image Enhacmenet: *Example 1*

The enhancement equals

$$EME_{0.88}(\hat{f}) - EME(f) = 19.61 - 17.11 = 2.50$$

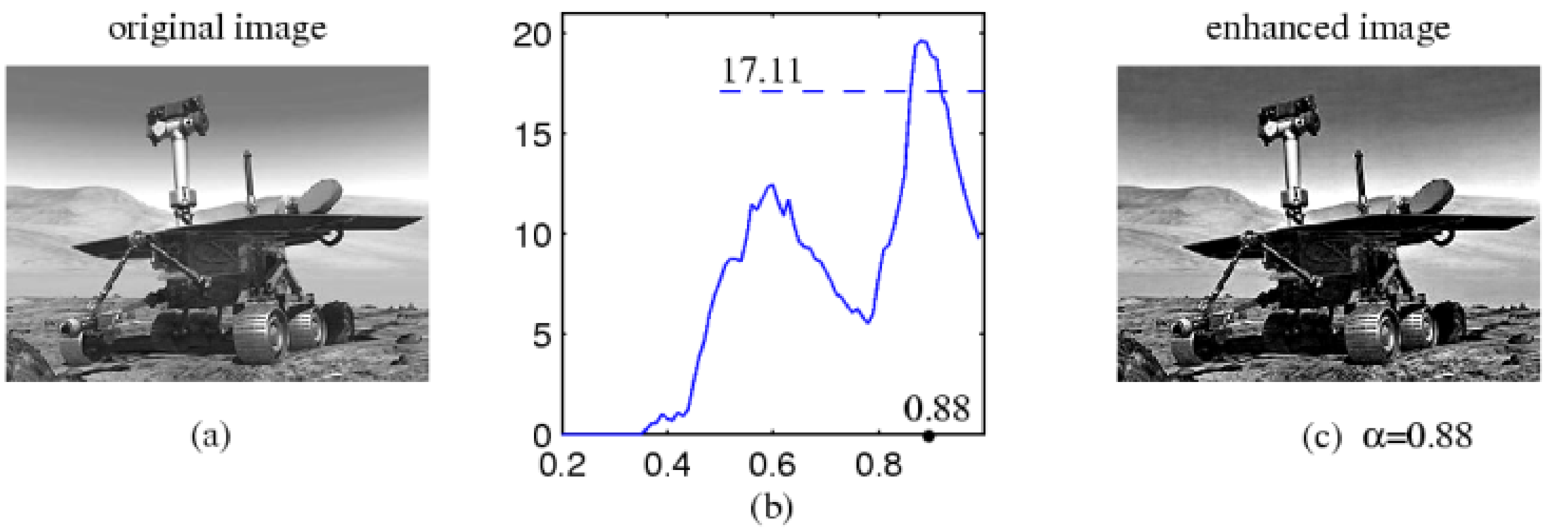


Fig. 2 (a) The original image, (b) the curve EME(α), and (c) the image enhanced by the α-rooting method.

Alpha-rooting by the 2-D DFT: *Example 2*

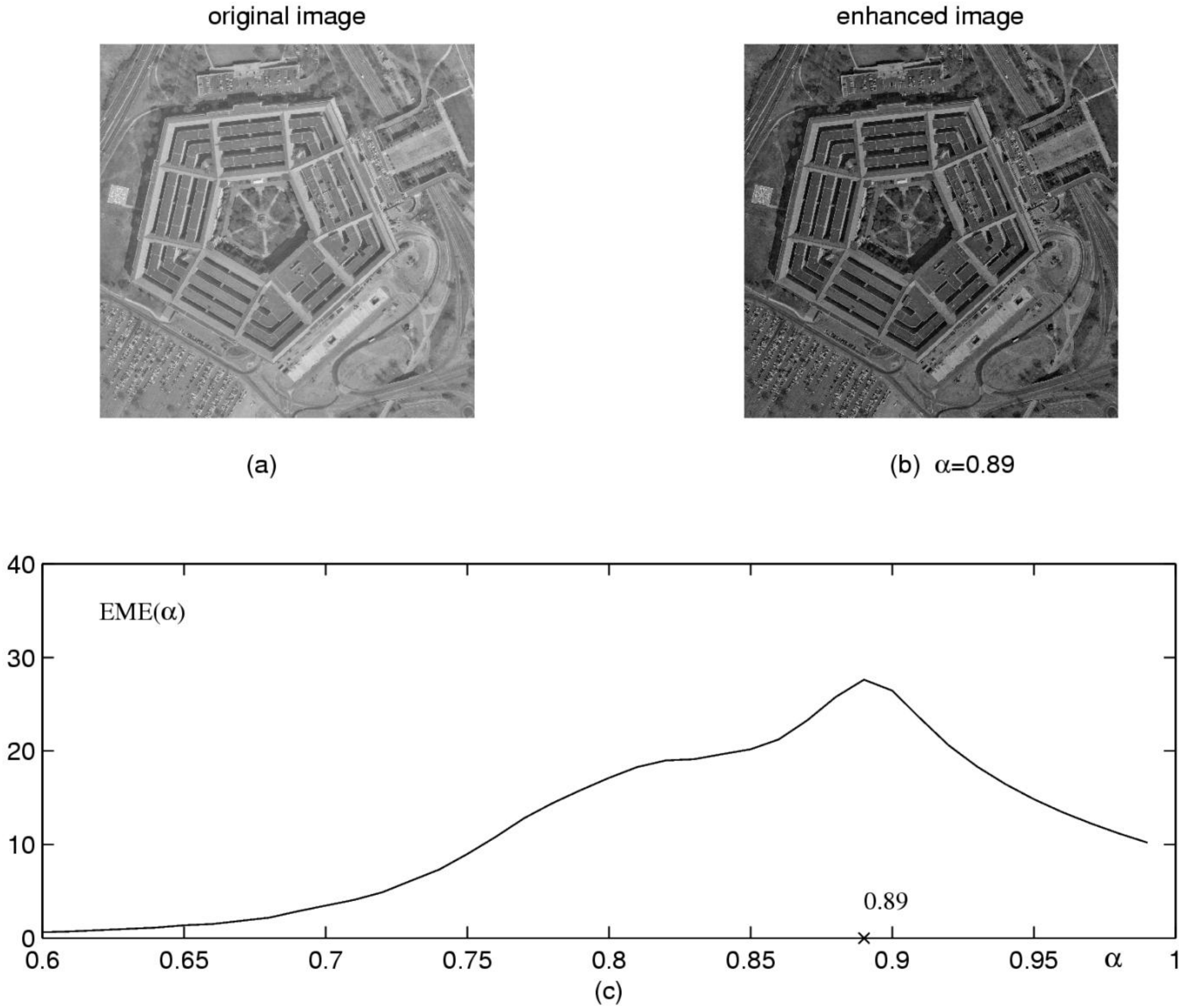


Fig. 3 Parameterized 2-D DFT image-enhancement (a) “pentagon” image 1024×1024, (b) the enhanced image, and (c) the curve of the enhancement measure calculated by 9×9 blocks.

Color / Quaternion Image Enhancement Measure

The images are in the RGB color space, $f_{n,m} \rightarrow \hat{f}_{n,m}$

$$f = (f_R, f_G, f_B) \text{ and } \hat{f} = (\hat{f}_R, \hat{f}_G, \hat{f}_B)$$

EME for color images is calculated by

$$EMEC(\hat{f}) = \frac{1}{k_1 k_2} \sum_{k=1}^{k_1} \sum_{l=1}^{k_2} 20 \log_{10} \left[\frac{\max_{k,l}\{\hat{f}_R, \hat{f}_G, \hat{f}_B\}}{\min_{k,l}\{\hat{f}_R, \hat{f}_G, \hat{f}_B\}} \right].$$

Here, the maximum and minimum values of the image in the (k,l)-th block are calculated as

$$\max(\hat{f}) = \max(\hat{f}_R, \hat{f}_G, \hat{f}_B) \text{ and } \min(\hat{f}) = \min(\hat{f}_R, \hat{f}_G, \hat{f}_B).$$

For four-component quaternion images, the EMEC is calculated similarly (we can call it EMEQ).

Right-side 2-D Quaternion Discrete Fourier Transform

Let $q_{n,m}$ be the quaternion discrete image of size $N \times N$

μ be the pure unit quaternion, $\mu^2 = -1$.

The right-side 2-D QDFT is defined as

$$Q_{p,s} = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} q_{n,m} W_\mu^{np+ms}, \quad p, s = 0 : (N-1).$$

where

$$W_\mu = W_{\mu;N} = \exp(-2\pi\mu/N)$$

The inverse right-side 2-D QDFT is calculated by

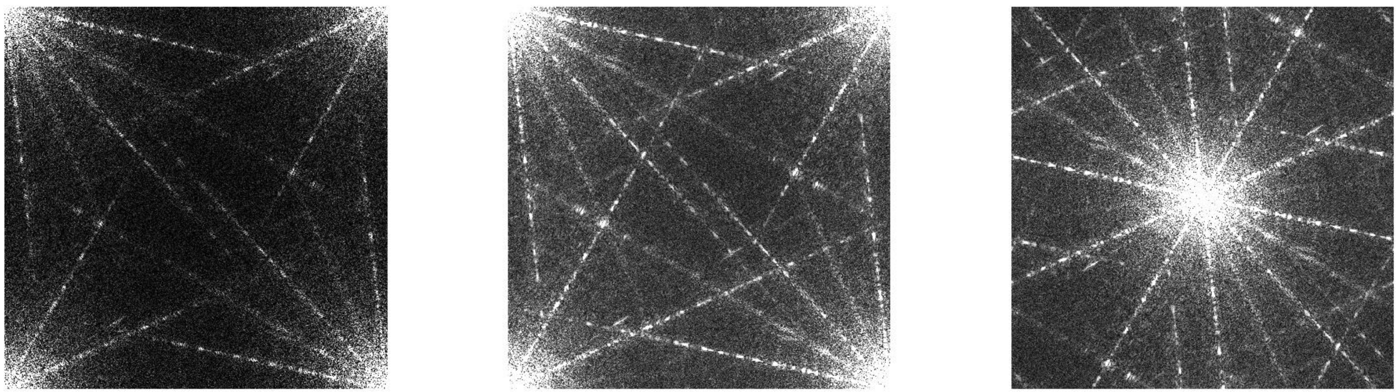
$$q_{n,m} = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{s=0}^{N-1} Q_{p,s} W_\mu^{-(np+ms)}, \quad n, m = 0 : (N-1).$$

Right-side 2-D Quaternion Discrete Fourier Transform

Example: 512×512 quaternion "pentagon" image

$$Q_{p,s} = (Q_{p,s})_e + \Big(i(Q_{p,s})_i + j(Q_{p,s})_j + k(Q_{p,s})_k\Big), \quad p,s = 0:511$$

The 512×512 2-D QDFT



(a) (b) (c)

Fig. 4 Magnitudes of (a) real component, $|Q_{p,s})_e|$, and imaginary component, $\sqrt{(Q_{p,s})_i^2 + (Q_{p,s})_j^2 + (Q_{p,s})_k^2}$ before (b) and after (c) cyclicly shifting to the center.

Alpha-rooting 2-D Quaternion DFT: *Example 2* …

Figure 5 shows the graph of the EMEC measure as a function of α with the same optimal value 0.89, for the same example with the quaternion "pentagon" image.

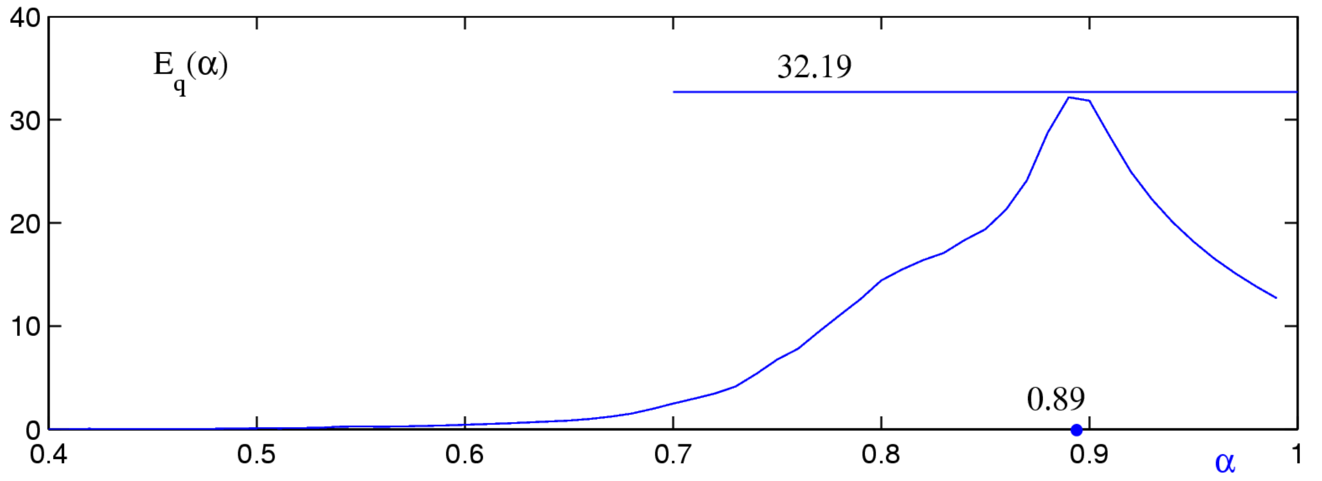


Fig. 5 The EMEQ of the alpha-rooting by the 2-D QDFT.

The curve of the enhancement measure EMEQ was calculated by 9×9 blocks.

Image enhancement: Examples

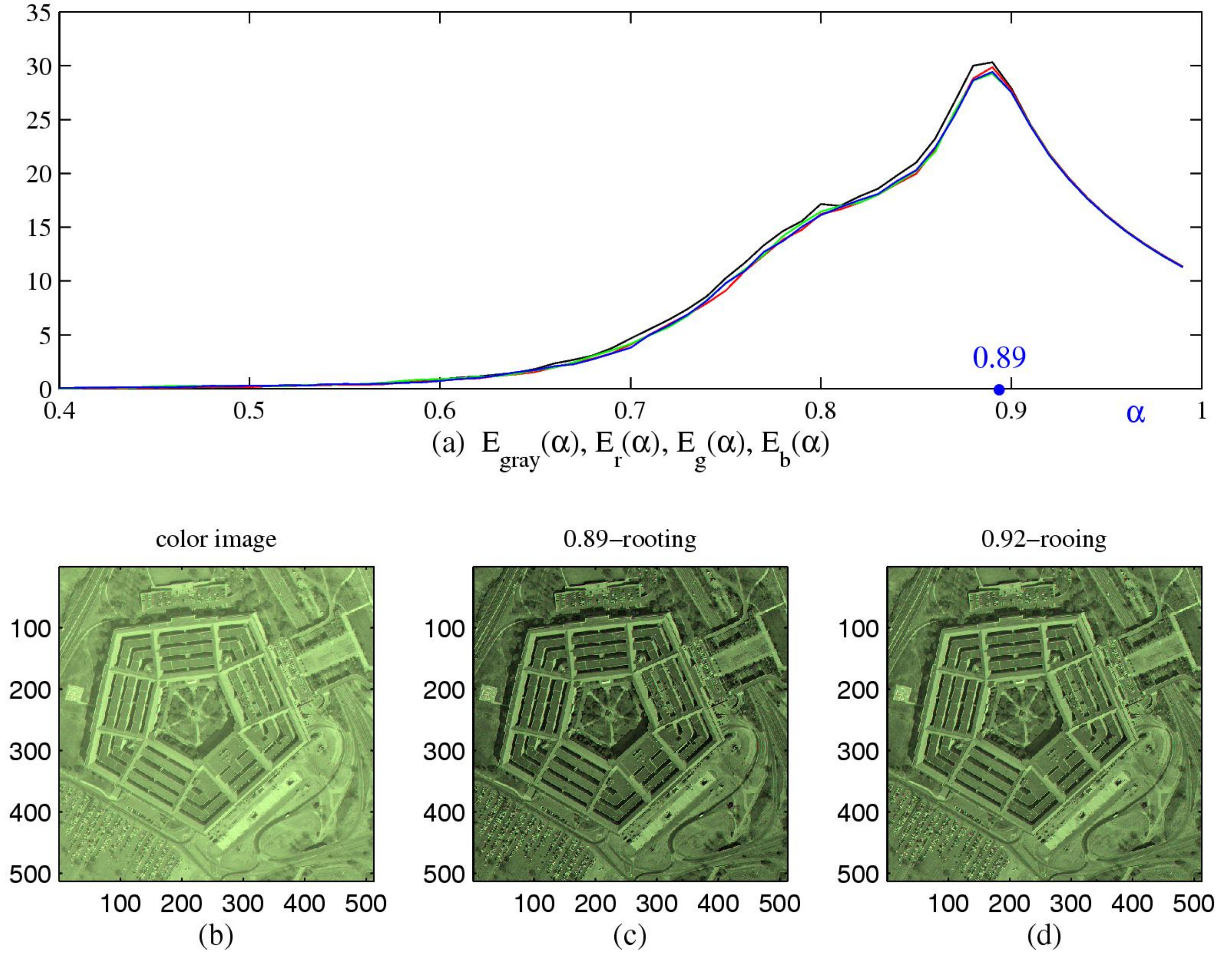


Fig. 6 (a) Enhancement functions for three channels of the tree image. (b,c) The α-rooting by the 2-D QDFT.

Image enhancement: *Example*

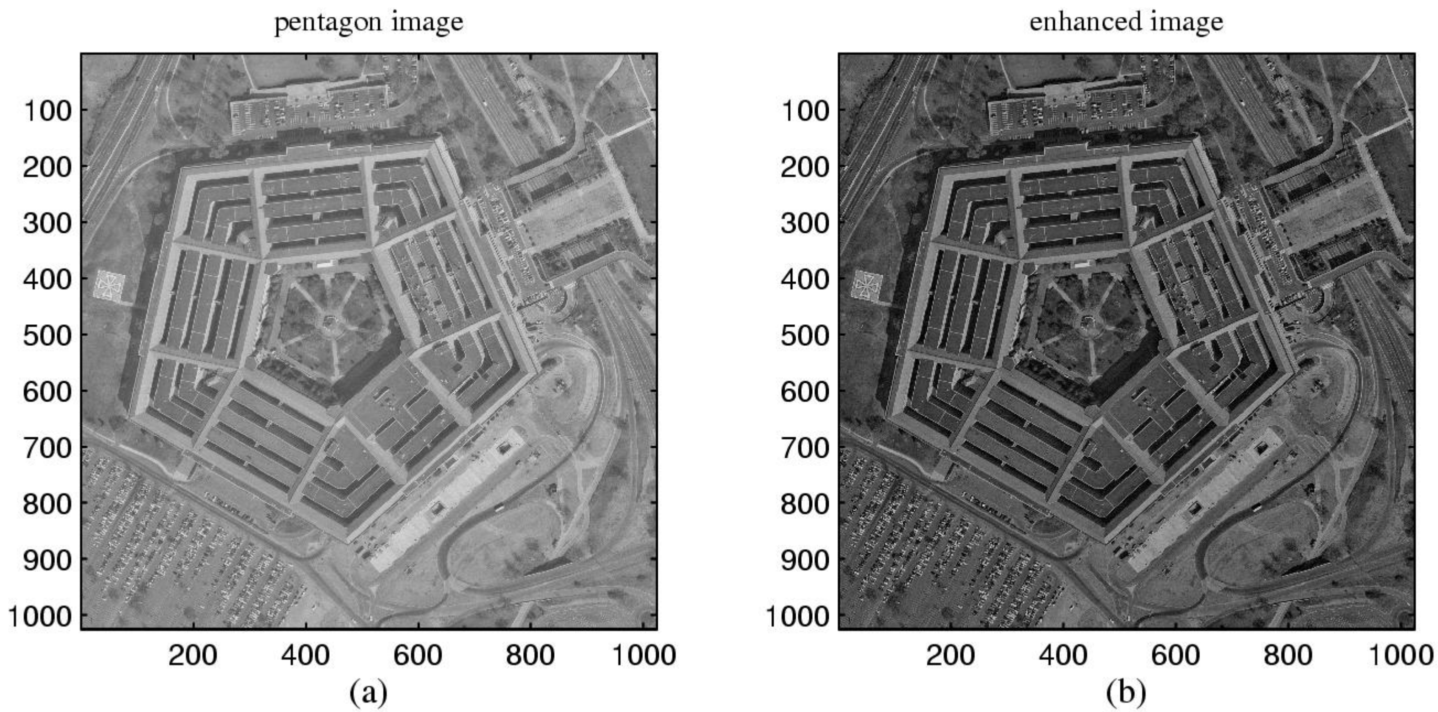


Fig. 7 (a) The original 1024× 1024 gray-scale “pentagon” image and (b) the enhanced image after processing by the alpha-rooting in the quaternion space.

Image enhancement: Example

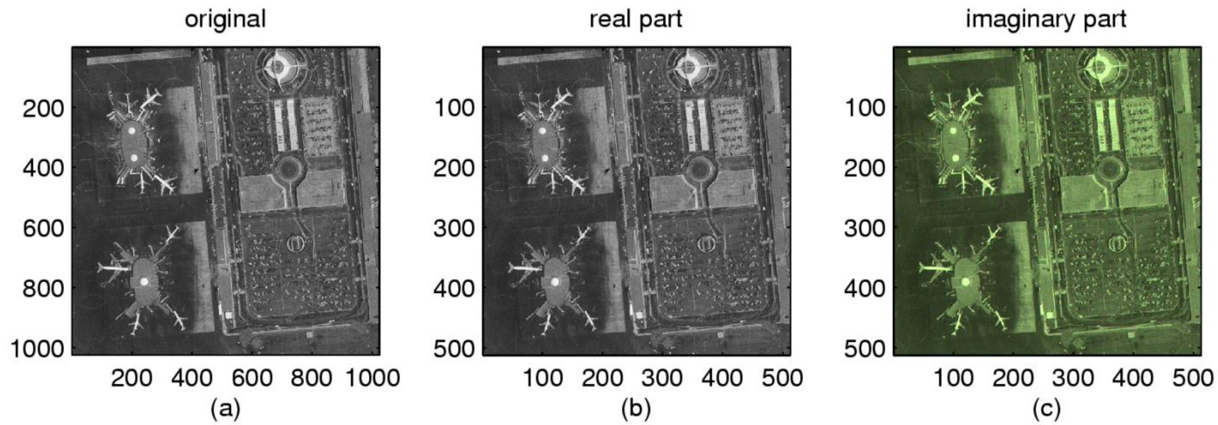


Fig. 8 (a) The original image "5.3.02" and (b,c) the quaternion image.

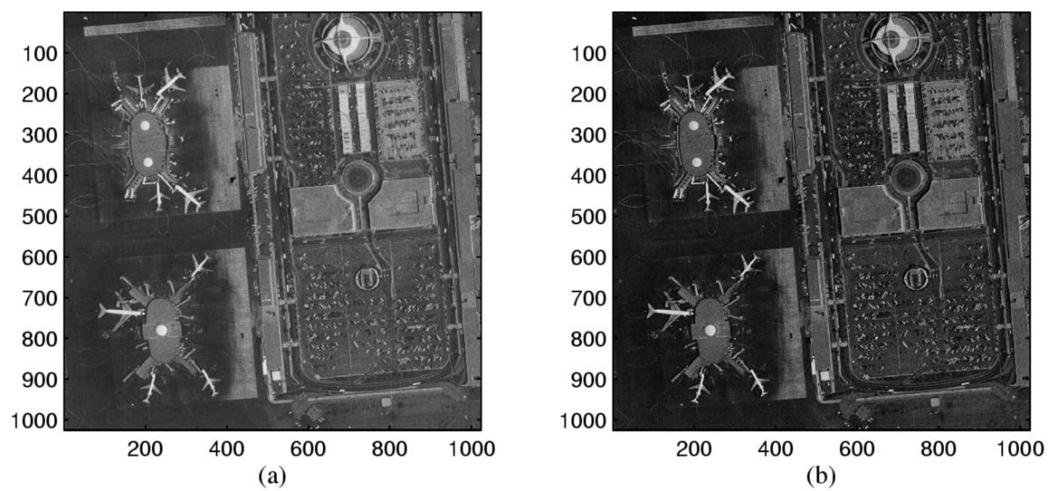


Fig. 9 (a) The original image and (b) the enhancement by α-rooting in the quaternion space (by using the 2-D QDFT).

Image enhancement: Example

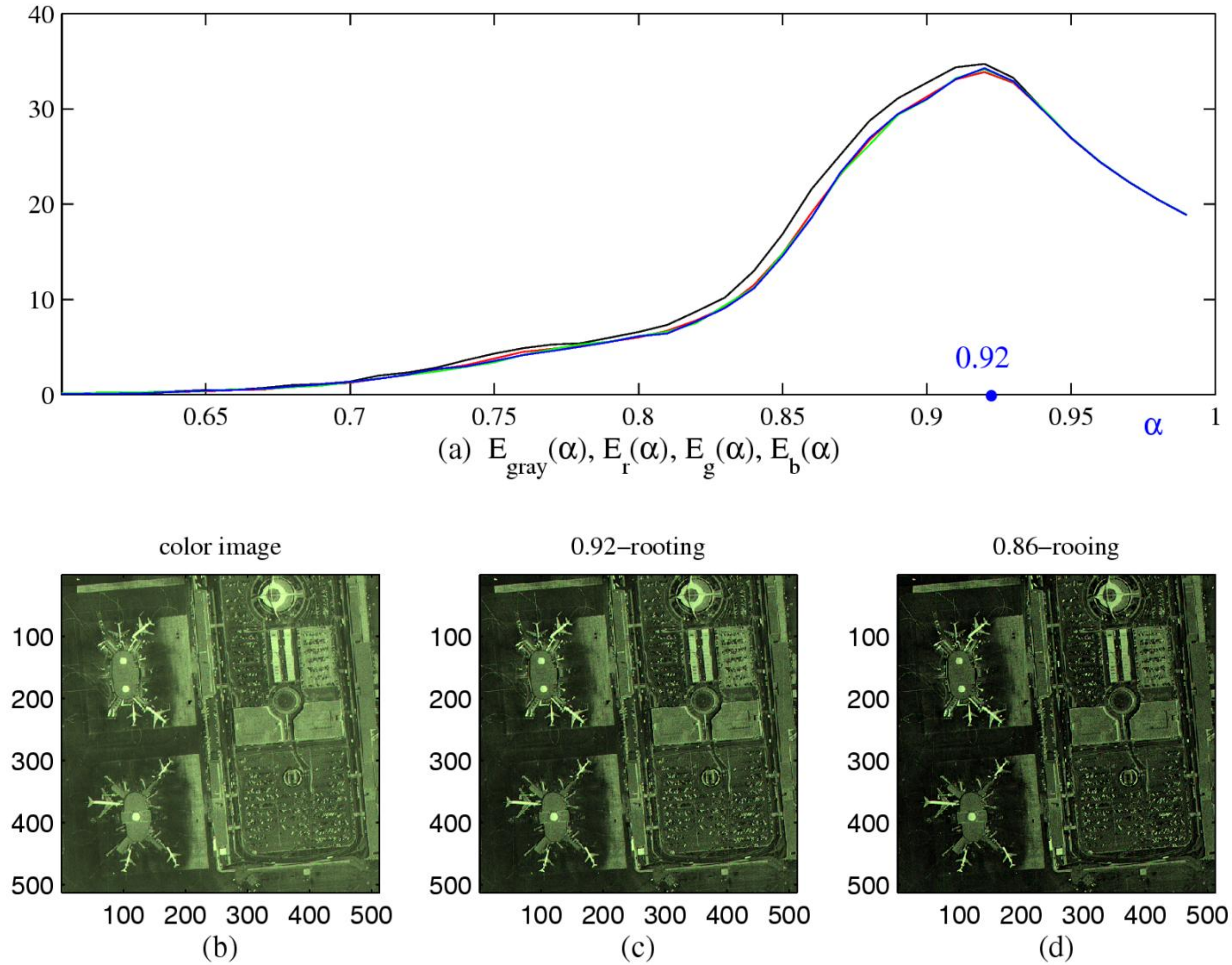


Fig. 10 (a) Four EME measures of the α-rooting, and the imaginary parts of (b) the quaternion "pentagon" image, and the images enhanced by (c) the 0.92-rooting and (d) the 0.86-rooting.

Summary

- New approach of image enhancement is proposed for gray-scale images, which is based on the idea of transforming the image into the quaternion space, where the image can be enhanced and filtered by using the concept of the 2-D QDFT.
- The enhancement by alpha-rooting by 2-D QDFT is described.
- Preliminary results show that the application of the 2-D QDFT plus the alpha-rooting method can be effectively used for enhancing gray-scale images.

References

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[2] A.M. Grigoryan and S.S. Agaian, "Retolling of color imaging in the quaternion algebra," *Applied Mathematics and Sciences: An International Journal (MathSJ)*, vol. 1, no. 3, pp. 23-39, 2014.

[3] A.M. Grigoryan, S.S. Agaian, "Tensor transform-based quaternion Fourier transform algorithm," *Information Sciences*, vol. 320, pp. 62-74, November 2015