

Asymmetric and Symmetric Gradient Operators  
with Application in Face Recognition  
in Renaissance Portrait Art

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## Abstract

- This paper proposes a class of gradient invertible asymmetric and symmetric operators. Examples of generated  $5 \times 5$  gradient operators in different directions are described.
- Extensive computer simulation is directed on 270 Renaissance portraits, including the art work of Raphael, Michelangelo, Leonardo Da Vinci and others.
- The simulation results show that the fusion of local binary patterns (LBP) with asymmetric and symmetric operators are better than traditional LBP features for face recognition, including Renaissance portraits the proposed.

## Asymmetric 3x3 Gradients of 2<sup>nd</sup> Order

We consider the Prewitt gradient in X-direction with the matrix

$$[P_x^2] = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 3 & -1 \\ 0 & 0 & -1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} -1 & 0 & 0 \\ -1 & 3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad (1)$$

These matrices describe the average differences of the image at pixel  $(n, m)$  with its three neighbors from the right and left,

$$G_{\text{right}}(f)_{n,m} = \frac{1}{3} \left[ \begin{array}{c} (f_{n,m} - f_{n-1,m+1}) + (f_{n,m} - f_{n,m+1}) \\ + (f_{n,m} - f_{n+1,m+1}) \end{array} \right],$$

$$G_{\text{left}}(f)_{n,m} = \frac{1}{3} \left[ \begin{array}{c} (f_{n,m} - f_{n-1,m-1}) + (f_{n,m} - f_{n,m-1}) \\ + (f_{n,m} - f_{n+1,m-1}) \end{array} \right].$$

The Prewitt gradient is the difference of these two gradients,

$$P_x^2(f) = G_{\text{right}}(f)_{n,m} - G_{\text{left}}(f)_{n,m}. \quad (2)$$

Similarly, the matrix of the Sobel gradient operator can be written as

$$[S_x^2] = \frac{1}{4} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 4 & -2 \\ 0 & 0 & -1 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} -1 & 0 & 0 \\ -2 & 4 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

The Sobel gradient is also the difference of two gradients, as many other known gradient operators.

**Question 1.** It is interesting to consider 2<sup>nd</sup> order gradients with different type left and right gradient operators. Can such mixed gradients be used in edge detection? For example, we consider the gradients with the left Prewitt and the right Sobel gradient matrices:

$$[G_x^2] = \frac{1}{4} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 4 & -2 \\ 0 & 0 & -1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} -1 & 0 & 0 \\ -1 & 3 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 4 & 0 & -3 \\ 4 & 0 & -6 \\ 4 & 0 & -3 \end{bmatrix}. \quad (3)$$

Note that a symmetry in the matrix is broken.

We call the asymmetric gradient with this matrix *the Sobel-Prewitt asymmetric gradient* in *X*-direction.

Example 1: The color  $672 \times 457$  pixel image of Leonardo Da Vinci's painting “Portrait of a Young Man (Portrait of the Musician Franchino Guffurio?)” and the Sobel-Prewitt asymmetric square-root gradient image calculated at each pixel by



(a) Original



(b) Grayscale



(c) Square-root operator

**Figure 1:** The color original image ‘leonardo10.JPG’ (from <http://www.abcgallery.com/>)

## *Gradients with 8 directions*

Many gradient images are calculated by using only two gradients; in the horizontal and vertical directions. In other words, these directions are defined by the angles  $0^\circ$  and  $180^\circ$  to the horizontal line. The diagonal and other directions can also be considered and added when calculating the gradient image for edge detection.

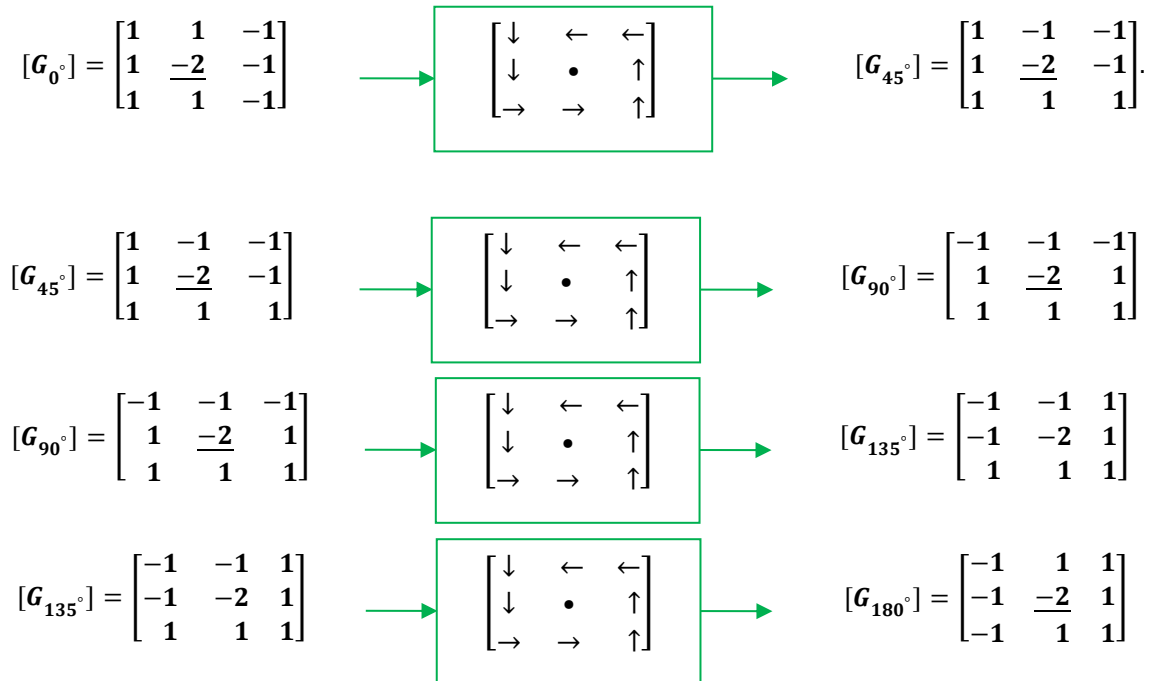
***Method of Matrix Rotation:*** The rotation of any  $3 \times 3$  matrix around its center is considered to be the following operation:

$$\begin{array}{c}
 \left[ \begin{array}{ccc}
 \downarrow & \leftarrow & \leftarrow \\
 \downarrow & \bullet & \uparrow \\
 \rightarrow & \rightarrow & \uparrow
 \end{array} \right] : \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & \underline{b_2} & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_2 & a_3 & b_3 \\ a_1 & \underline{b_2} & c_3 \\ b_1 & c_1 & c_2 \end{bmatrix}.
 \end{array}$$

*Counter clock-wise rotation by 45 degrees*

It is the counter clock-wise rotation of coefficients of the matrix by 45 degree.

The rotation of the Prewitt matrix that is denoted by  $G_0^\circ$ :



After rotating the matrix by the angles  $225^\circ$ ,  $270^\circ$ , and  $315^\circ$ , we obtain the following matrices:

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ -1 & -1 & -1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

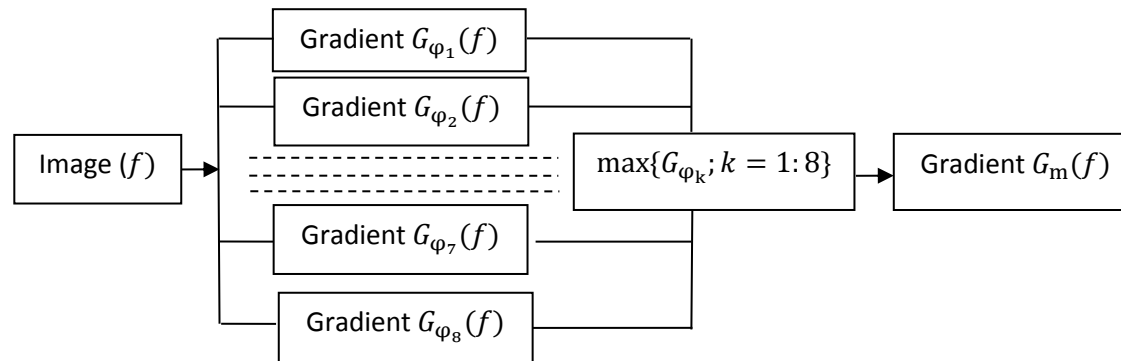


All eight matrices are different and the eight gradient images  $|G_{\varphi_k}(f)|$ ,  $k = 1:8$ , are different and correspond to the set of angles

$$\Psi_8 = \{\varphi_k, k = 1:8\} = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\}.$$

The concepts of the square-root, maximum, and magnitude gradient image can be defined by using the set of gradient images  $G_{\varphi_k}(f)$ . For instance; the maximum gradient image can be defined as

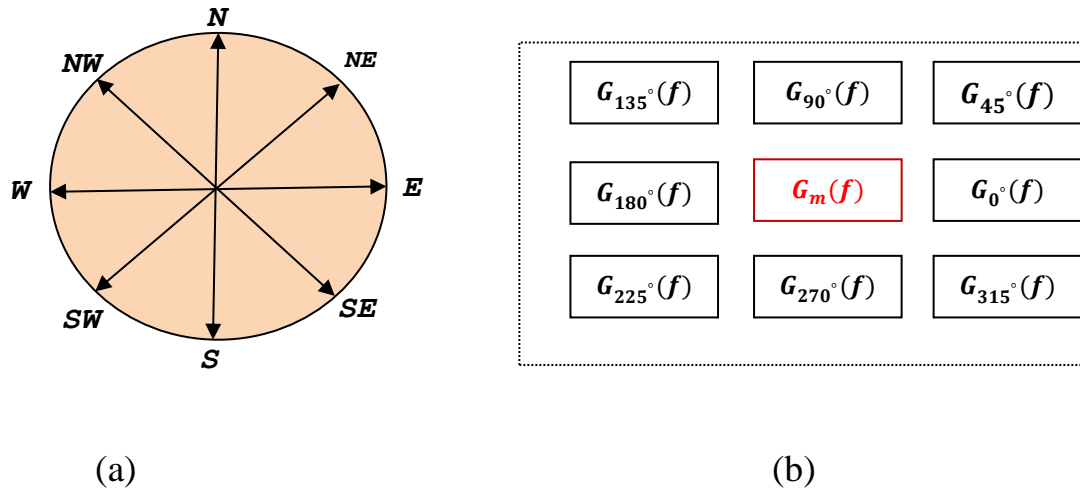
$$G_m(f) = \max\{|G_{\varphi_k}(f)|; k = 1:8\},$$



**Figure 2:** The diagram of calculation of the maximum gradient image.

The compass names can be given to these gradient matrices  $G_{\varphi_k}$ ,  $k = 1:8$ , that are East, Northeast, North, Northwest, West, Southwest, South, and Southeast, respectively.

These eight gradient images in the absolute scale and the maximum gradient image can be shown together in the order that corresponds to the compass directions, as shown in Fig. 3.



**Figure 3:** (a) Compass directions and (b) the maximum gradient image with all eight components.



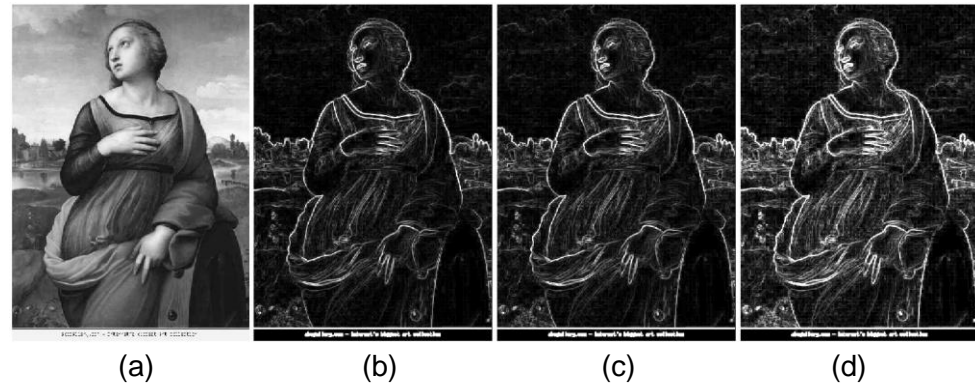
**Fig. 4:** The grayscale image of 'raphael27.jpg' (from <http://www.abcgallery.com/>) together with the eight gradient images  $|G_{\varphi_k}(f)|$ ,  $k = 1:8$ .

*The eight-neighbor Laplacian:* The square-root gradient image is defined as

$$G^2(f) = \frac{1}{\sqrt{8}} \sqrt{[G_{\varphi_1}(f)]^2 + [G_{\varphi_2}(f)]^2 + \dots + [G_{\varphi_8}(f)]^2}.$$

The magnitude gradient image is calculated by

$$G(f) = \frac{1}{8} [|G_{\varphi_1}(f)| + |G_{\varphi_2}(f)| + \dots + |G_{\varphi_8}(f)|].$$



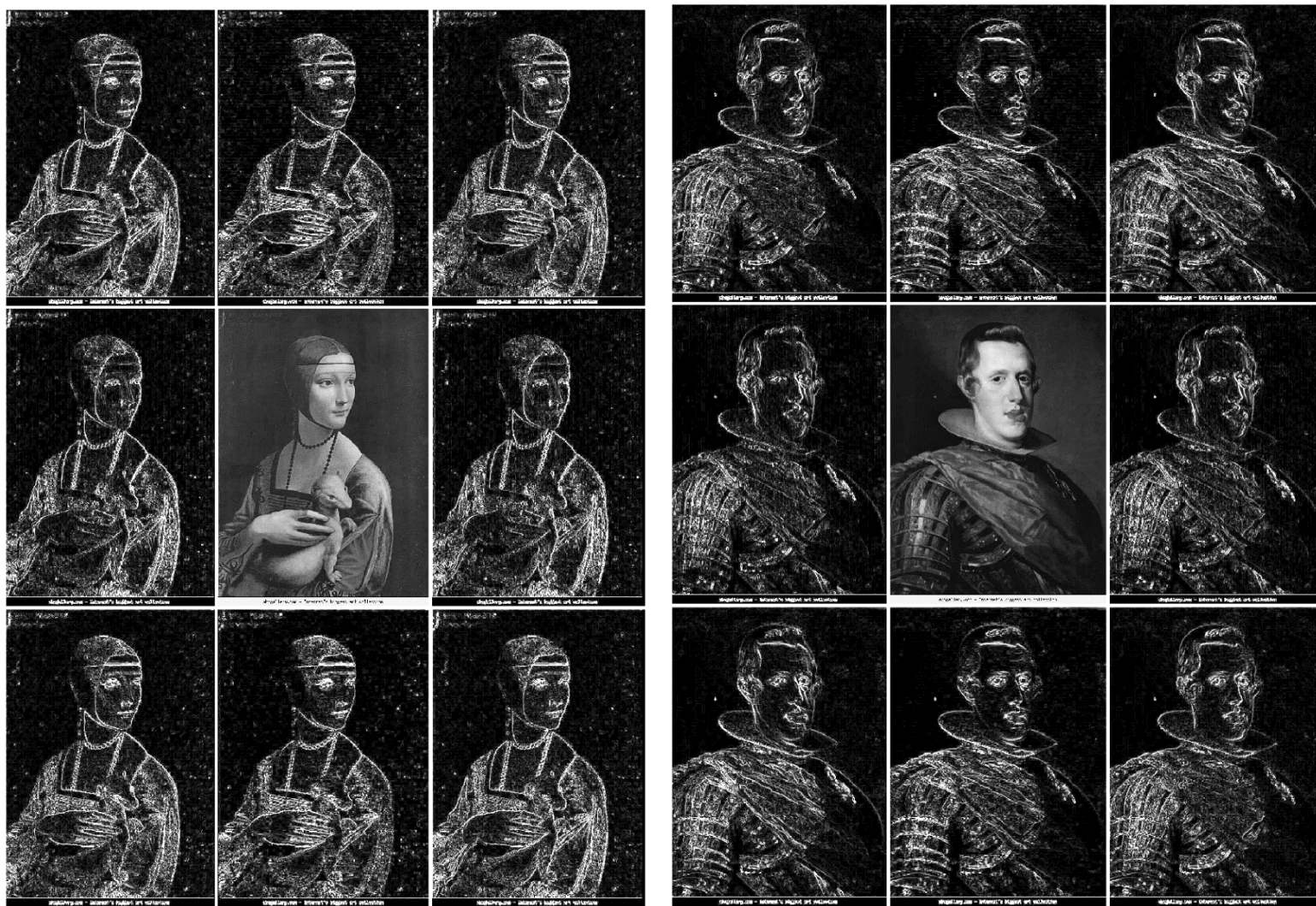
**Fig. 5:** (a) The grayscale of the color original image 'raphael27.jpg' (from <http://www.abcgallery.com/>), (b) the magnitude gradient image, (c) the square-root gradient image, and (d) the maximum magnitude gradient image.

In these figures, the grayscale 741×528 pixel image of Raphael's painting “St. Catherine” is shown together with all eight compass gradients (in Fig. 4) and gradient images (in Fig. 5).

All eight gradient images are different, since the matrices of rotations are different, and they are shown in Table 1.

$[G_{0^\circ}]$ $= \frac{1}{5} \begin{bmatrix} 1 & 1 & -1 \\ 1 & \underline{-2} & -1 \\ 1 & 1 & -1 \end{bmatrix}$	$[G_{45^\circ}]$ $= \frac{1}{5} \begin{bmatrix} 1 & -1 & -1 \\ 1 & \underline{-2} & -1 \\ 1 & 1 & 1 \end{bmatrix}$	$[G_{90^\circ}]$ $= \frac{1}{5} \begin{bmatrix} -1 & -1 & -1 \\ 1 & \underline{-2} & 1 \\ 1 & 1 & 1 \end{bmatrix}$
$[G_{135^\circ}]$ $= \frac{1}{5} \begin{bmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$[G_{180^\circ}]$ $= \frac{1}{5} \begin{bmatrix} -1 & 1 & 1 \\ -1 & \underline{-2} & 1 \\ -1 & 1 & 1 \end{bmatrix}$	$[G_{225^\circ}]$ $= \frac{1}{5} \begin{bmatrix} 1 & 1 & 1 \\ -1 & \underline{-2} & 1 \\ -1 & -1 & 1 \end{bmatrix}$
$[G_{270^\circ}]$ $= \frac{1}{5} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{-2} & 1 \\ -1 & -1 & -1 \end{bmatrix}$	$[G_{315^\circ}]$ $= \frac{1}{5} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{-2} & -1 \\ 1 & -1 & -1 \end{bmatrix}$	$\sum [G_{\varphi_k}]$ $= \frac{16}{5} \cdot \frac{1}{8} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{-8} & 1 \\ 1 & 1 & 1 \end{bmatrix}$

*Table 1: Matrices of gradient operators.*



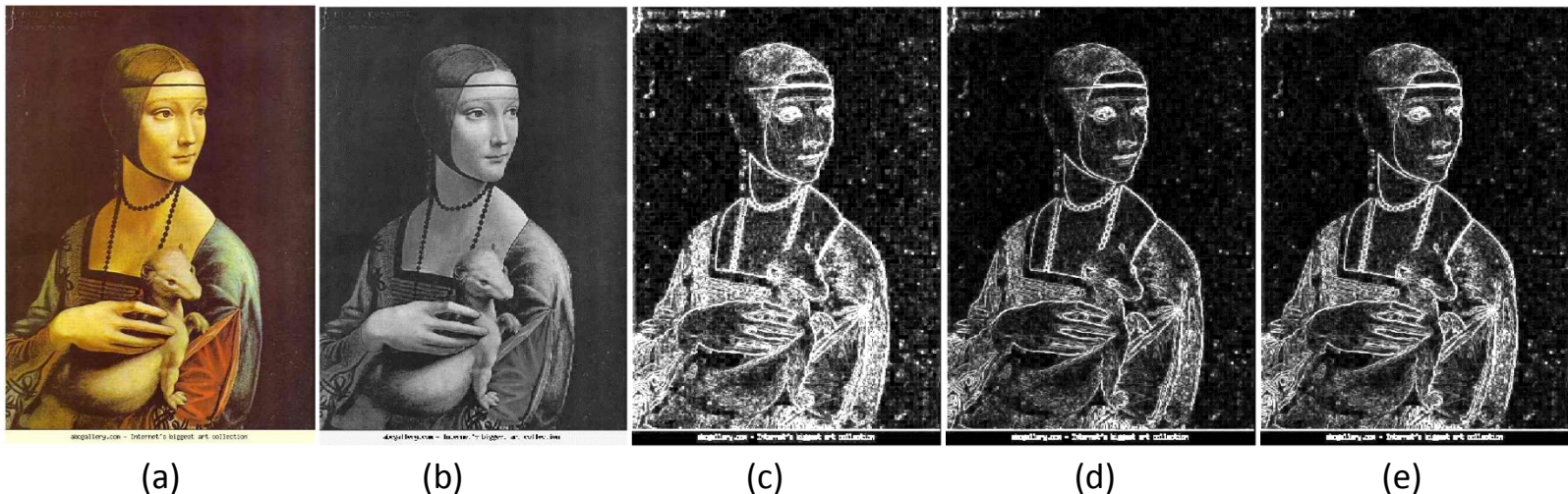
(a)

(b)

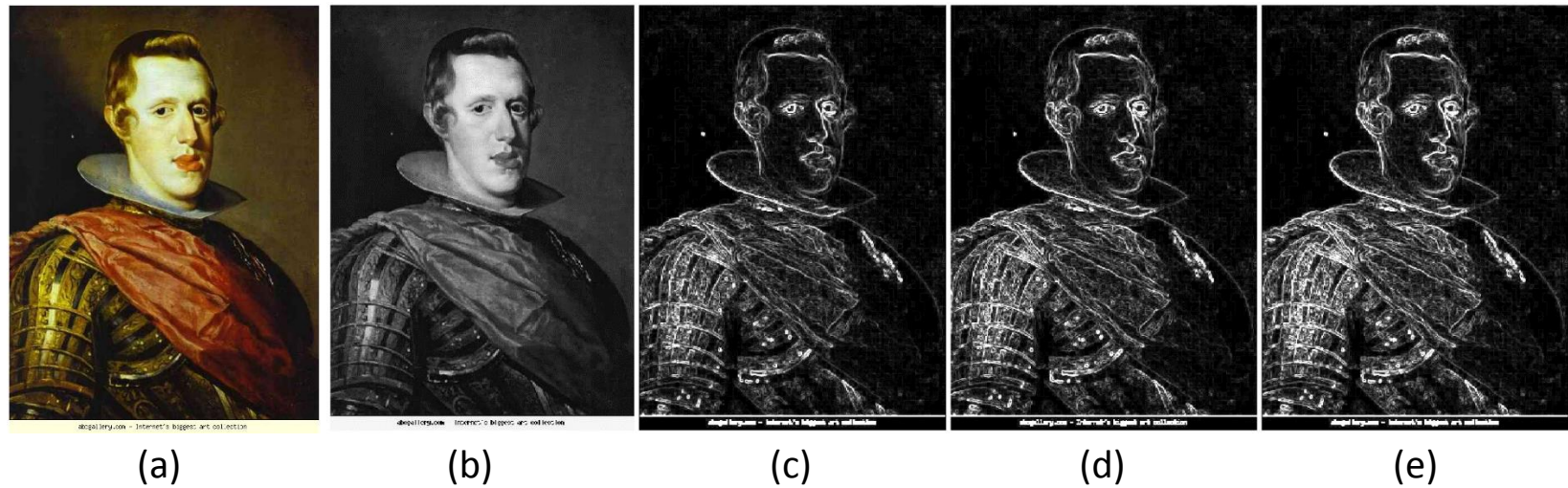
**Figure 6:** Leonardo Da Vinci's painting "Portrait of Cecilia Gallerani (Lady with an Ermine)" and Diego Velázquez's painting "Philip IV in Armour." The grayscale images (a) of 'leonardo9.jpg' and (b) 'velazquez21.jpg' in the center and their eight gradient images. (from <http://www.abcgallery.com/>).

The square-root and magnitude gradient images by the Art asymmetric gradient images are defined as

$$G^2(f) = \sqrt{\frac{1}{8} \sum_{k=0}^7 [G_{k45^\circ}(f)]^2} \quad \text{and} \quad G(f) = \frac{1}{8} \sum_{k=0}^7 |G_{k45^\circ}(f)|.$$



**Figure 8:** (a) The color image 'leonardo9.jpg' of the Leonardo Da Vinci's painting "Portrait of Cecilia Gallerani (Lady with an Ermine)" (from <http://www.abcgallery.com/>), (b) the grayscale image, (c) the maximum gradient image  $G_m(f)$ , the magnitude gradient image  $G(f)$ , and the square-root gradient image  $G^2(f)$ .



**Figure 8:** (a) The color image ‘velazquez21.jpg’ of Diego Velázquez’s painting “Philip IV in Armour” (from <http://www.abcgallery.com/>), (b) the grayscale image, (c) the maximum gradient  $G_m(f)$  image, (d) the magnitude gradient image  $G(f)$ , and (e) the square-root gradient image  $G^2(f)$ .

## Gradient operators with 4 directions

Similar sets of gradients can be composed by using other gradient operators that include the Sobel-Prewitt asymmetric operator, Sobel 5-level operator, the Prewitt 3-level operator, and the Kirsch operator with the following matrices [2], respectively:

$$\frac{1}{12} \begin{bmatrix} 4 & 0 & -3 \\ 4 & 0 & -6 \\ 4 & 0 & -3 \end{bmatrix}, \frac{1}{4} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}, \frac{1}{15} \begin{bmatrix} 5 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & -3 & -3 \end{bmatrix}.$$

**Sobel-Prewitt-matrix**    **(1,1,2,1|0)-matrix**    **(1,1,1,1|0)-matrix**    **(5/3,1,1,1|-1)-matrix**

It should be noted that not all eight gradient images  $|G_{\varphi_k}(f)|$ ,  $k = 1:8$ , may be different. In such cases, the set of gradients is smaller than eight.



For example, we consider the Sobel 5-level gradient operator in the horizontal direction with the matrix

$$[G_{0^\circ}] = [G_x^2] = \frac{1}{4} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}. \quad (12)$$

The sequential rotation of this matrix by  $45^\circ$  results in the matrices

$$[G_{45^\circ}] = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}, \quad [G_{90^\circ}] = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix},$$

$$[G_{135^\circ}] = \begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad [G_{180^\circ}] = -[G_{0^\circ}(f)],$$

$$[G_{225^\circ}] = -[G_{45^\circ}], \quad [G_{270^\circ}] = -[G_{90^\circ}], \quad [G_{315^\circ}] = -[G_{1135^\circ}].$$

Therefore, the gradients are considered only for the set of four angles  $\Psi = \{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$ : the compass directions, East, North East, North, and North West. The Sobel asymmetric maximum, square-root, and magnitude gradient images can be defined as

$$G_m(f) = \max\{|G_{\varphi_k}(f)|; k = 1:4\},$$

$$G^2(f) = \sqrt{\frac{1}{4} \sum_{k=1}^4 [G_{\varphi_k}(f)]^2}, \quad G(f) = \frac{1}{4} \sum_{k=1}^4 |G_{\varphi_k}(f)|,$$

respectively. The sum of four gradient matrices is

$$\sum_{k=1}^4 [G_{\varphi_k}] = \frac{1}{2} \begin{bmatrix} -1 & -2 & -2 \\ 1 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} = 3 \cdot \underbrace{\frac{1}{6} \begin{bmatrix} -1 & -2 & -2 \\ 1 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix}}.$$

*\*The Sobel-Prewitt asymmetric gradient operator defines the compass set with eight angles.*

## The Art Gradient Operators with 8 Directions

The Art gradient asymmetric operator in the  $X$ -direction is

$$[A] = \frac{1}{8} \begin{bmatrix} 2 & -2 & -1 \\ 4 & 0 & -2 \\ 2 & -2 & -1 \end{bmatrix}. \quad (14)$$

To compare with the Sobel 5-level gradient, we can write that

$$\frac{1}{8} \begin{bmatrix} 2 & -2 & -1 \\ 4 & 0 & -2 \\ 2 & -2 & -1 \end{bmatrix} = \frac{1}{2} \cdot \left( \underbrace{\frac{1}{4} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}}_{\text{Sobel X}} + \underbrace{\frac{1}{4} \begin{bmatrix} 1 & -2 & 0 \\ 2 & 0 & 0 \\ 1 & -2 & 0 \end{bmatrix}}_{\text{New Operator}} \right).$$

A new gradient operator is added to the Sobel operator.

$$\frac{1}{4} \begin{bmatrix} 1 & -2 & 0 \\ 2 & 0 & 0 \\ 1 & -2 & 0 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 & 0 \\ 0 & 0 & 0 \\ 2 & -2 & 0 \end{bmatrix} + \frac{1}{2} \cdot \frac{1}{2} \begin{bmatrix} -1 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

\*It is the sum of differencing from the left operators in the  $X$ - and  $Y$ -directions.

All eight matrices of rotations by the angles  $\varphi_k$  of the set  $\Psi = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\}$  are different (Table 2).

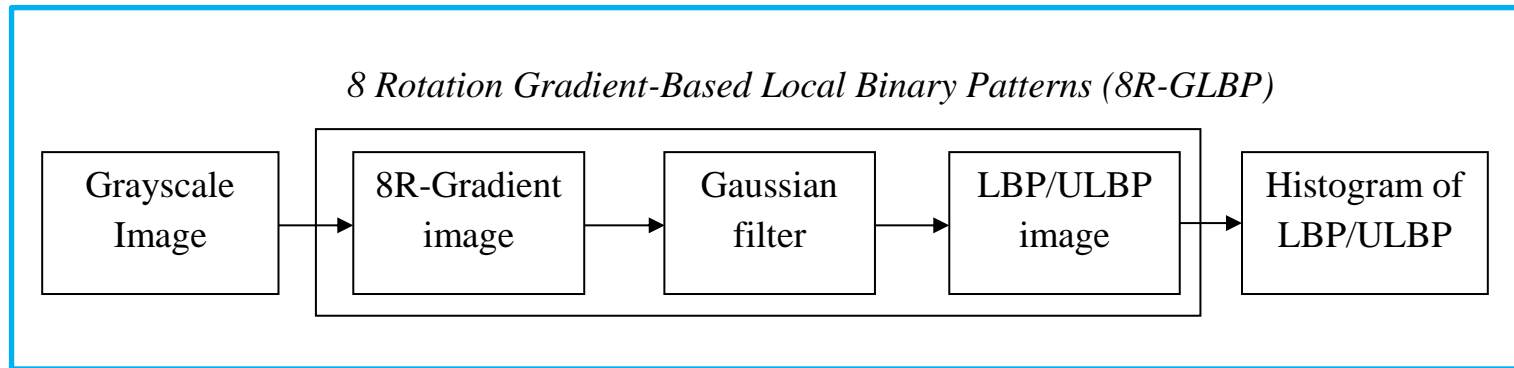
$[G_{0^\circ}]$ $= \frac{1}{8} \begin{bmatrix} 2 & -2 & -1 \\ 4 & 0 & -2 \\ 2 & -2 & -1 \end{bmatrix}$	$[G_{45^\circ}]$ $= \frac{1}{8} \begin{bmatrix} -2 & -1 & -2 \\ 2 & 0 & -1 \\ 4 & 2 & -2 \end{bmatrix}$	$[G_{90^\circ}]$ $= \frac{1}{8} \begin{bmatrix} -1 & -2 & -1 \\ -2 & 0 & -2 \\ 2 & 4 & 2 \end{bmatrix}$
$[G_{135^\circ}]$ $= \frac{1}{8} \begin{bmatrix} -2 & -1 & -2 \\ -1 & 0 & 2 \\ -2 & 2 & 4 \end{bmatrix}$	$[G_{180^\circ}]$ $= \frac{1}{8} \begin{bmatrix} -1 & -2 & 2 \\ -2 & 0 & 4 \\ -1 & -2 & 2 \end{bmatrix}$	$[G_{225^\circ}]$ $= \frac{1}{8} \begin{bmatrix} -2 & 2 & 4 \\ -1 & 0 & 2 \\ -2 & -1 & -2 \end{bmatrix}$
$[G_{270^\circ}]$ $= \frac{1}{8} \begin{bmatrix} 2 & 4 & 2 \\ -2 & 0 & -2 \\ -1 & -2 & -1 \end{bmatrix}$	$[G_{315^\circ}]$ $= \frac{1}{8} \begin{bmatrix} 4 & 2 & -2 \\ 2 & 0 & -1 \\ -2 & -1 & -2 \end{bmatrix}$	$\sum [G_{\varphi_k}]$ $= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

*Table 2: Matrices of gradient operators.*

## Portrait Image Representation for Recognition

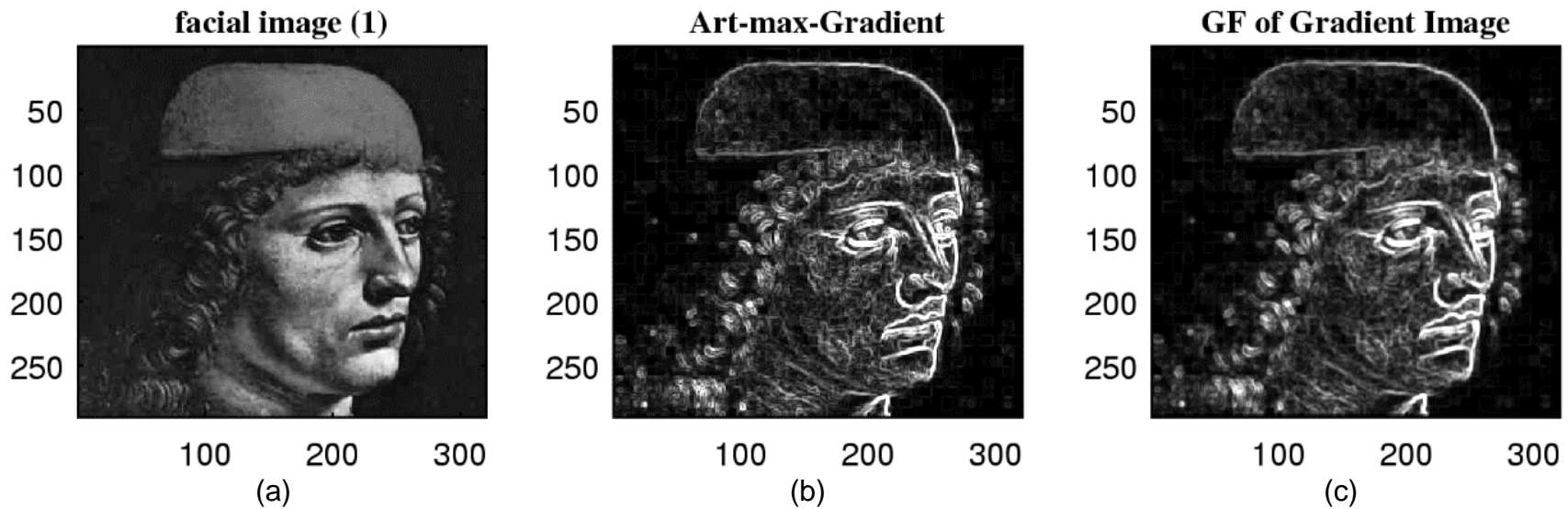
We consider the grayscale facial image representation, or description, which is based on the local binary patterns (LBP) over the whole facial image [1,2]. We describe this representation in terms of simple gradient operators with following composition of the 8-bit LBP image and its histogram, which can be used as the feature in classification of facial images.

The main parts of representation of the grayscale  $N \times M$ -pixel facial image  $f_{n,m}$  are shown in the block-diagram of Fig. 9.



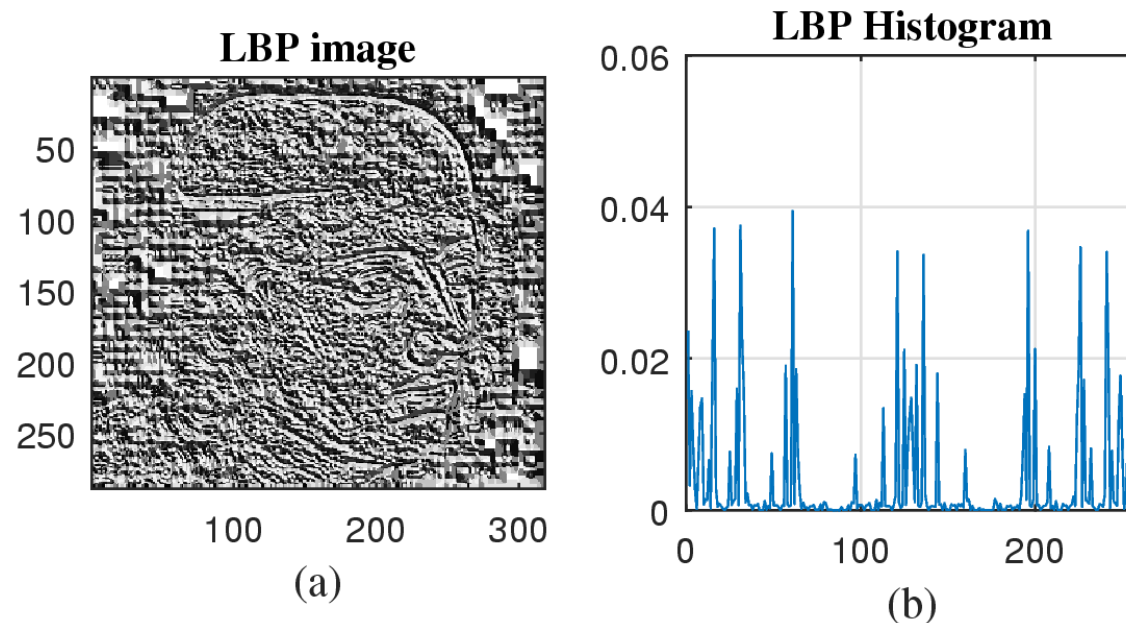
**Figure 9.** The block-diagram of the facial image representation.

As an example, we consider the portrait of the man from image of the Leonardo Da Vinci's painting “Portrait of a Young Man” shown in Fig. 1. The  $290 \times 320$  pixel face image is shown in Fig. 10 in part (a). The Art gradient maximum image is shown in part (b), and after filtering by the 2-D Gaussian function with standard deviation  $1/2$  in part (c).



**Figure 10.** (a) The grayscale face of the image ‘leonardo10.jpg’ (from <http://www.abcgallery.com/>), and the maximum gradient by eight Art asymmetric operators before (b) and after (c) filtration by the Gaussian filter.

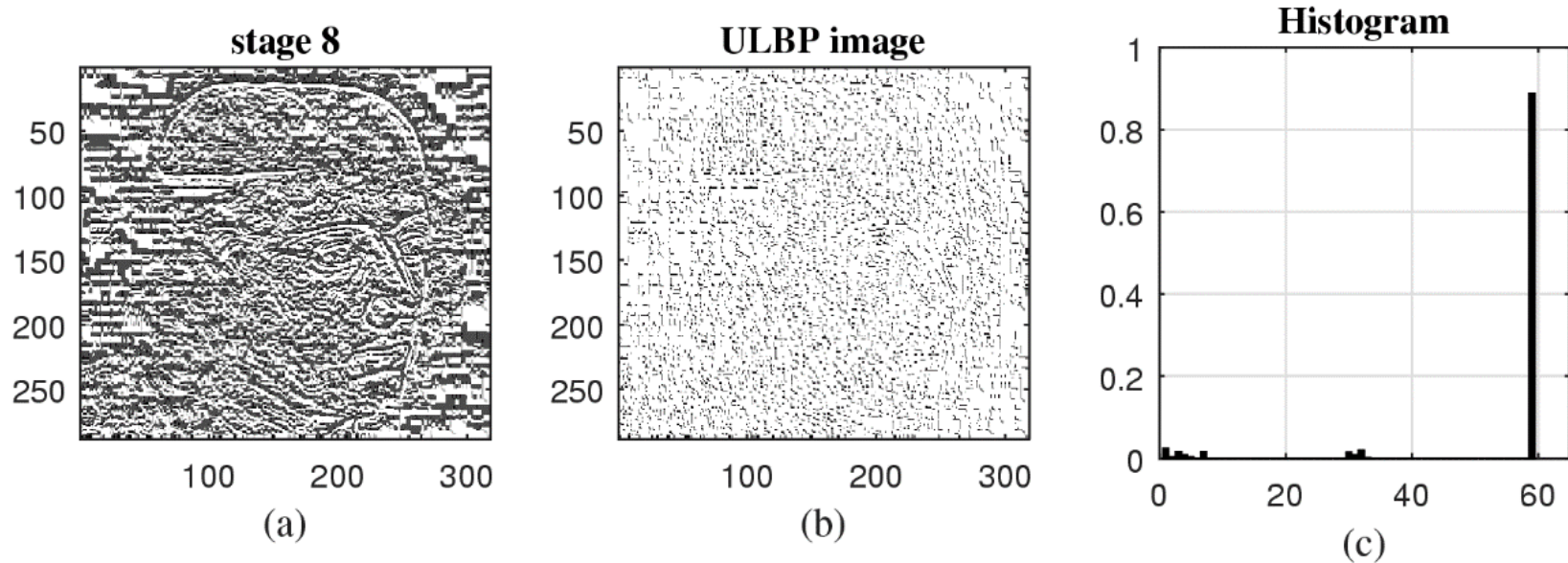
Figure 11 shows the LBP image of this face in part (a) and its histogram with the range [0,255] in part (b).



**Figure 11.** (a) The local binary pattern image and (b) and the histogram of the image.

The concept of the uniform LBP was introduced to reduce the range of the histogram from [0,255] to [0,58]. The window for calculating the local pattern is  $3 \times 3$  and the ULBP is calculated recursively in 8 stages, by using the special mapping to the patterns in number 59.

Figure 12 shows the final stage of the mapping in part (a) and the ULBP of the portrait image in part (b). The histogram of this image is shown in part (c).

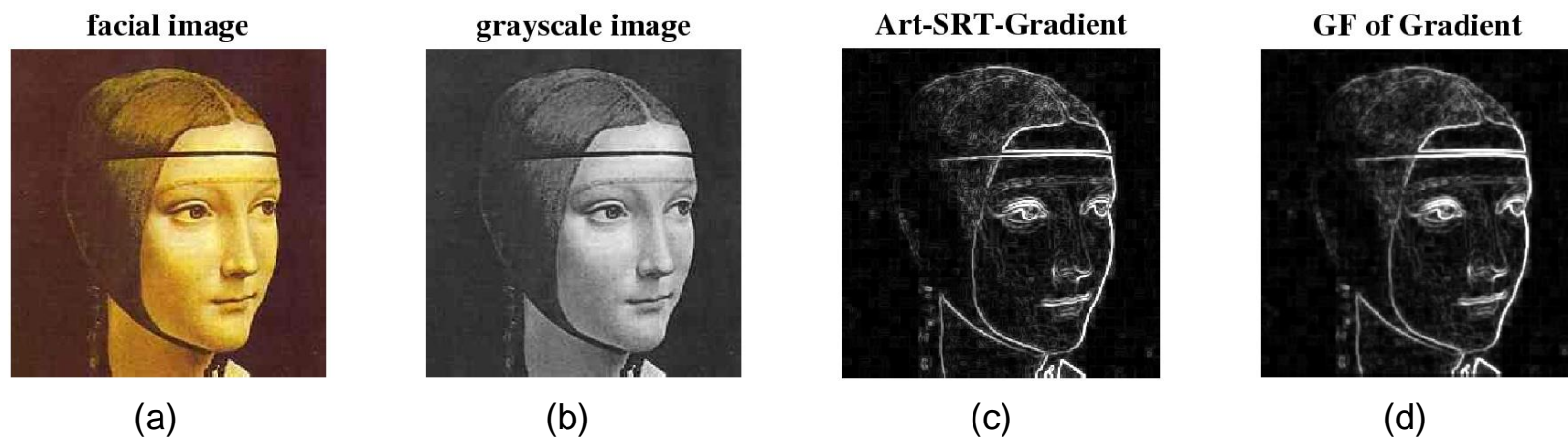


**Figure 12.** The uniform local binary pattern image (a) before and (b) after the mapping, and (c) the histogram of this image.



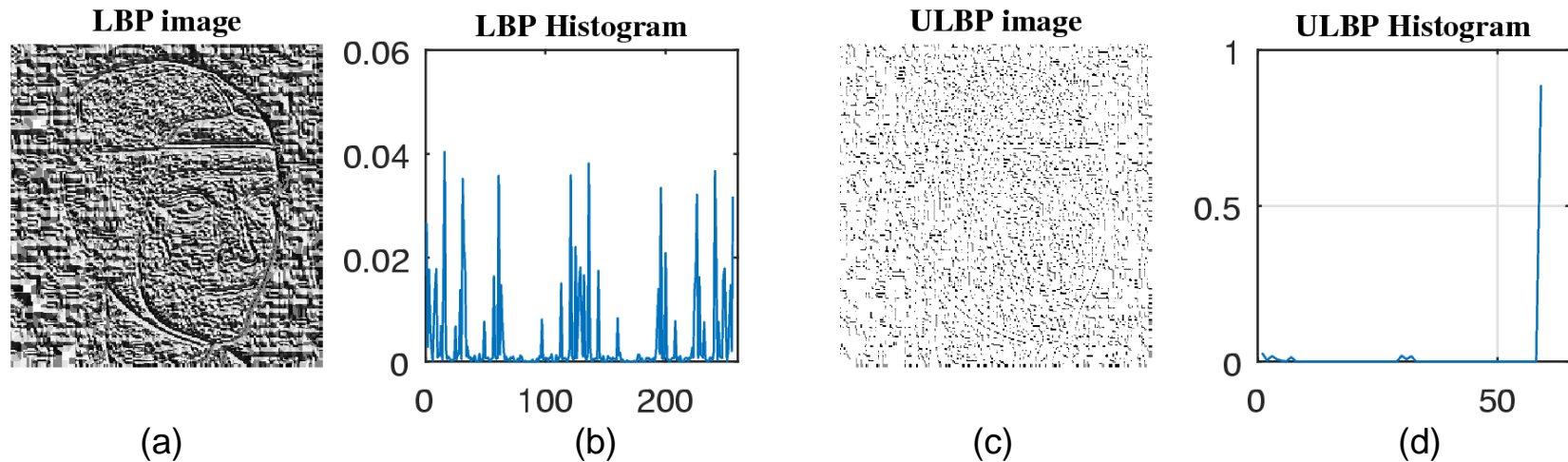
Together with the maximum gradient calculated by eight directions, the square-root and magnitude gradients can also be used in extracting the characteristic of the image in form of the histogram.

Figure 13 shows the  $270 \times 260$  color pixel face image of the Leonardo Da Vinci's painting "Portrait of Cecilia Gallerani (Lady with an Ermine)" in part (a) and its grayscale image in (b). The square-root Art gradient image is show in part (c) and in part (d) after filtering by the Gaussian function with the standard deviation 0.5.



**Figure 13.** (a) The grayscale face of the image 'leonardo9.jpg' (from <http://www.abcgallery.com/>), and the square-root gradient by eight Art asymmetric operators before (b) and after (c) filtration by the Gaussian filter.

The ULBP of the same face image is shown in part (b) and its histogram in part (d).



**Figure 14.** (a) The local binary pattern image and (b) its histogram, (c) the uniform local binary pattern image and (d) its histogram.

It should be noted that in all experimental results shown above, the gradient images were calculated and further processed without any thresholding which is a complicated operation and only changes slightly the histogram of the LBP and ULBP images, which effect much of the results of the face recognition.

## Summary

A novel face recognition approach is proposed, by using multiple feature fusion across color, spatial and frequency domains. The proposed approach is useful and applicable not only for face recognition, but also for object recognition.

We are planning to evaluate the presented face recognition concept, by using the color FERET database: <http://www.face-rec.org/databases/>.

## References

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3. A.M. Grigoryan, S.S. Agaian, "Two general models for gradient operators in imaging," Proceedings of IS&T International Symposium, Electronic Imaging: Algorithms and Systems, 28 Jan.-2 Feb., Burlingame, CA, 2018. ...