The Novel Arc Low-Pass Filter

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Abstract

In this paper, we propose a new form of the amplitude response for designing low-pass filters, that is defined by a modified arctangent function in the frequency domain. This is a parameterized and very smooth function which can be used for any tolerance scheme with given specifications. The filter depends on two parameters that allow us to straighten the magnitude response from the passband to the stopband without increasing the computational complexity of the filter. The main properties of the low-pass filter with the proposed magnitude response, which we call the Arc low-pass filter, are considered and compared with the well known Chebyshev and Butterworth filters.

Purpose

Development of a novel lowpass filter based on the arctangent function and comparison with the Chebyshev and Butterworth lowpass filters.

Introduction

In digital signal processing, we are faced with the problem of designing a lowpass filter with the cutoff frequency in the frequency domain. For instance, when recovering the Fourier transform of the original continuous-time signal from its sampled version, the ideal filter is considered. The ideal filter is not causal and therefore, cannot be utilized as a filter. This paves the way for the use of different classes of causal Butterworth and Chebyshev filters in real-time filtering applications as these filters have gradual transitions from the passband to stopband. In this paper, we propose a new frequency response function for low-pass filter design and compare it with the current standard methods of approximation of ideal filters, namely, the Butterworth and Chebyshev filters. Unlike the Chebyshev filter, the proposed low-pass filter, *the Arc low-pass filter*, does not produce ripples and exhibits a steeper roll-off at the cutoff frequency in comparison to both standards.

Definition and Properties

This filter is based on the arctangent-like frequency function which can closely fit any rectangular window, by specific parameterization of the arctangent function. The Arc lowpass filter still maintains the ratio of the input and output signal to be 0.5 while the parameter is not restricted to being an integer and can take the form of any non-integer value, thereby improving the flatness of the output signal in the passband and minimizing the amplitude of the signal in the stopband. The realization of the Arc filter is simple and makes it attractive for practical design.

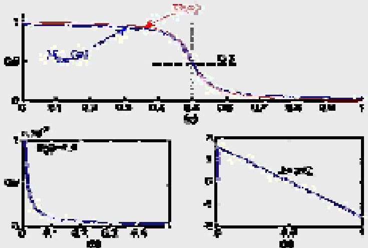
1. Arc Low-pass Filter

The Arc low-pass filter is defined by the following amplitude response,

$$H_{Arc}(\omega) = H_{\alpha,\beta}(\omega) = \frac{1}{2}\left(1-\tan^{-1}\left(\left(\frac{\omega}{\omega_{cut}}-\beta\right)\alpha\pi\right)\right)$$

$$H_{Arc}(\omega) = H_{Arc}(-\omega)$$

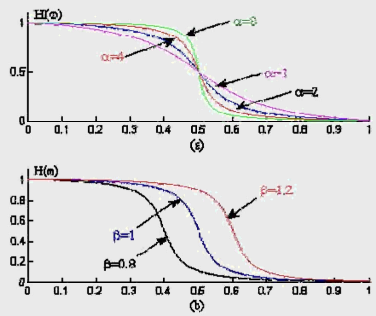
As an example, Figure 1 part a shows the function $H(\omega)$, for cutoff frequency ω_{cut} = 0.5rad/s and parameters β = 1 and α = 4. The amplitude of the inverse Fourier transform (IFT) of the function at frequencies ω of the interval [0, 0.5] is given in b, and the phase of the transform in c. The phase at t = 0 equals zero, and is linear in the rest of the interval.



Thus, the complete formula for calculation of the amplitude response can be written as

$$H_{Arc}(\omega) = \frac{\tan^{-1}\left(\left(\frac{1}{\omega_{cut}}-\beta\right)\alpha\pi\right)-\tan^{-1}\left(\left(\frac{\omega}{\omega_{cut}}-\beta\right)\alpha\pi\right)}{\tan^{-1}(\beta\alpha\pi)+\tan^{-1}\left(\left(\frac{1}{\omega_{cut}}-\beta\right)\alpha\pi\right)}$$

With the value of β held constant, if we let $\alpha \to \infty$, the value of the normalizing constants, A and B go to 1 and 0 respectively. This property is used in attaining α when certain filter design parameters such as ω_{stop} , ω_{cut} , $1-\delta_1$, and δ_2 are given.



We can modify the amplitude response of the Arc filter, to achieve zero derivative at the origin, by defining the following amplitude response

$$H_{Arc}(\omega) = H_{\alpha,\beta}(\omega) = \frac{1}{2}\left(1-\tan^{-1}\left(\left(\left(\frac{\omega}{\omega_{cut}}\right)^{2n}-\beta\right)\alpha\pi\right)\right), \qquad \omega \in [-1,1]$$

The derivate of this response when n = 1 equals

$$H'_{Arc}(\omega) = -\frac{\alpha\pi\omega}{\omega_{cut}}\frac{1}{1+\left(\left(\frac{\omega}{\omega_{cut}}\right)^2-\beta\right)^2\alpha^2\pi^2}$$

and we see that $H'_{Arc}(0)=0$. This property of the derivate of the modified response is similar to that exhibited by Butterworth filter.

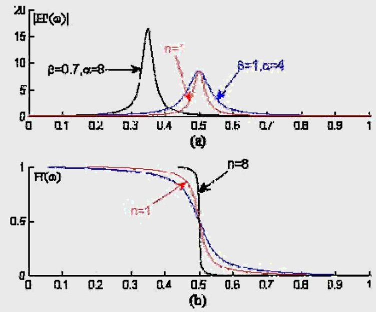


Fig. 3. (a) Derivative of the amplitude response $H(\omega)$ for β = 1, α = 4, and β = 0.7, α = 8, and (b) amplitude responses $H(\omega)$ for β = 1, α = 4, and $H_n(\omega)$ for n = 1 and 8.

2. Properties and Comparison

If the passband gain $r_{dB} = -20\log_{10}(1-\delta_1)$ is known, then one can find the minimal value of α for the magnitude response $H(\omega)$ that fits any prescribed tolerance scheme. From the condition $H(\omega_{cut}) = 1-\delta_1$ and $H(\beta\omega_{cut}) = \frac{A}{2}+B$,we obtain

$$\alpha = \frac{1}{\pi}\frac{\tan D_1(\delta_1)}{(1-\beta)} \quad \text{where} \quad D_1(\delta_1) = 1-\frac{2(10^{-\frac{r_{dB}}{20}}-B)}{A}$$

To find the α , when the parameter is given for the amplitude response to be less than δ_2 in the stopband$(\omega_{stop},1)$,we can perform the following calculations

$$\alpha = \frac{1}{\pi}\frac{\tan D_2(\delta_2)}{\left(\frac{\omega_{stop}}{\omega_{cut}}-\beta\right)} \quad \text{where} \quad D_2(\delta_2) = 1-\frac{2(\delta_2-B)}{A}$$

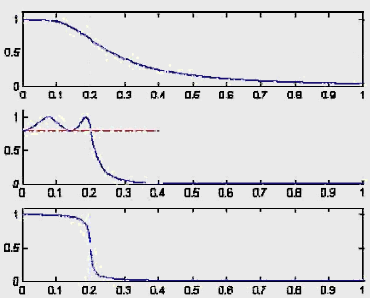


Fig. 4. Amplitude responses of the Butterworth, Chebyshev, and arc low-pass filters.

Experimental Results

Our preliminary results show that Arc low-pass filter can be used for filtering signals from noise. The ideal lowpass filter of length 128 has been used. Results of filtration are referred to as the real parts of the inverse DFTs.

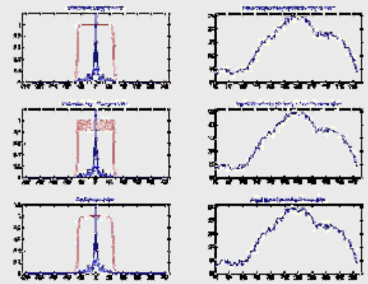


Fig. 5. Amplitude spectrums of the signal filtered by the low-pass (a) Butterworth, (c) Chebyshev, and (e) Arc filters, and (b, d, f) results of the inverse DFTs.

In order to compare the low-pass filters mentioned above, we consider the tolerance scheme with the following specifications: the cutoff is at 64Hz, the transition region from passband to stopband equals [61, 68], the ripples equal r_{dB} = 1.9682dB, and the maximum value of the ripples in the stopband is 0.2dB. The Butterworth filter with order n = 17 and the Chebyshev filter of type I of order n = 7 satisfy the specifications of this tolerance scheme as shown in the figure. The Arc filter for this tolerance scheme is calculated with parameters α = 7 and β = 1.0094. The root-mean-square error of signal filtration for the Butterworth filter equals 0.0021, for the Chebyshev filter equals 0.0051, and 0.0017 for the Arc filter.

Conclusions

The Arc low-pass filter with the amplitude response defined by the arctangent function in the proposed form allows us to fit any tolerance scheme without increasing the computational complexity of the calculation. This filter is parameterized and is a very smooth function in both the passband and stopband. We believe that the Arc low-pass filter can be effectively used in practice for processing 1-D and 2-D signals.

References

[1] B.S. Heck and E.W. Kamen, *Fundamentals of Signals and Systems*, Prentice Hall, Upper Saddle River, NJ, 2000.
[2] A.V. Oppenheim and R.W Schafer, *Discrete Time Digital Signal Processing*, Prentice Hall, Englewood Cliffs, NJ, 1989.
[3] C.L. Phillips, J.M. Parr, and E.A. Riskin, *Signals, Systems and Transforms*, Prentice Hall, Upper Saddle River, NJ, 2003.
[4] C. Jack, *Digital Signal Processing: An Overview of Basic Principles*, New Jersey: Prentice Hall, 2000.

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