Fast Signal-Induced Transforms in Image Enhancement

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Outline

- Introduction
- Parameterized DsiHT
- Angular representation and Image Enhancement
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- Experimental Results
- Conclusions

Abstract

The concept of the discrete unitary transforms induced by given signals is modified and developed.

The basic transformations composing such transforms are parameterized and the energy (or its partial part) of signals is transferred to one of their components in different paths. We focus on the case when all basic transformations themselves represent Givens rotations of the inputs or modified inputs.

Applications of proposed angular transforms for image enhancement are described.

Introduction

It is very important to define a complete system of functions that carry much information of processed signals. Such an example is the system of eigenfunctions for the KL transform.

Recently, a simple model of composing the discrete signal-induced heap transforms (DsiHT) has been introduced. Complete systems of the DsiHT are referred to as waves generated by input signals.

We here consider the modified DsiHT and its unique angular form, when the signal is described as an inverse angular transform.

Discrete signal-induced heap transform (DsiHT)

Given a vector-generator $\mathbf{x} = (x_0, x_1, ..., x_{N-1})'$, the N-point DsiHT of a signal $\mathbf{z} = (z_0, z_1, ..., z_{N-1})$ is defined by $N-1$ basic transformations

$$\begin{aligned} T(\varphi_k) : \mathbf{z} \rightarrow (z_0, ..., z_{k_1-1}, f(z_{k_1}, z_{k_2}, \varphi_k), z_{k_1+1}, ..., \\ z_{k_2-1}, g(z_{k_1}, z_{k_2}, \varphi_k), z_{k_2+1}, ..., z_{N-1}) \end{aligned} \tag{1}$$

where $k = 1 : N-1$, and the pair of numbers (k_1, k_2) is uniquely defined by k.

We consider the case, when $k_1 = 0$ and $k_2 = k$. In other words, the path P of processing components of the signal starts from $k_2 = 1$ and consequently continues until $N-1$.

The value of z_0 is renewed on each stage of calculation.

Discrete signal-induced heap transform (DsiHT)

The parameter φ_k is defined from the second equation, or *the angular equation* of the system of decision equations

$$\begin{cases} f(x_0, x_{k_2}, \varphi_k) = y_0 \\ g(x_0, x_{k_2}, \varphi_k) = a_k \end{cases} \tag{2}$$

for a given parameter a_k.

The value of y_0 is calculated and considered to be a new value of x_0 for the next stage of calculations.

System of equations (2) is solving for a vector-generator $\mathbf{x} = (x_0, x_1, ..., x_{N-1})$.

Example 1: The four-point DsiHT

The DsiHT by the vector-generator $\mathbf{x} = (x_0, x_1, x_2, x_3)'$ is defined in matrix form as

$$\mathbf{y} = \mathbf{T}\mathbf{x} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1\mathbf{x} = \begin{bmatrix} c_3 & 0 & 0 & -s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_3 & 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ 0 & 1 & 0 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

$c_k = \cos(\varphi_k),\ \ s_k = \sin(\varphi_k),\ \varphi_k = -\tan^{-1}(x_k/y_0)\,,\ k = 1, 2, 3.$

$y_0 = x_0$, for $k = 1$,
$y_0 = y_0^{(2)} = (\mathbf{T}_1\mathbf{x})_0$,
$y_0 = y_0^{(3)} = (\mathbf{T}_2\mathbf{T}_1\mathbf{x})_0$.

Example 1: The four-point DsiHT

The matrix of the $\mathbf{x} = (1,2,3,4)'$-vector induced DsiHT

$$\mathbf{T} = \begin{bmatrix} 0.6831 & 0 & 0 & 0.7303 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.7303 & 0 & 0 & 0.6831 \end{bmatrix} \begin{bmatrix} 0.5976 & 0 & 0.8018 & 0 \\ 0 & 1 & 0 & 0 \\ -0.8018 & 0 & 0.5976 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\times \begin{bmatrix} 0.4472 & 0.8944 & 0 & 0 \\ -0.8944 & 0.4472 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Its output $\mathbf{y} = (5.4772, 0, 0, 0)'$ is calculated as

$$\mathbf{y} = \mathbf{T}\mathbf{x} = \begin{bmatrix} 0.1826 & 0.3651 & 0.5477 & 0.7303 \\ -0.8944 & 0.4472 & 0 & 0 \\ -0.3586 & -0.7171 & 0.5976 & 0 \\ -0.1952 & -0.3904 & -0.5855 & 0.6831 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

Angles $\varphi_1 = -1.1071$, $\varphi_2 = -0.9303$, and $\varphi_3 = -0.8188$.
The number $5.4772^2 = 30$ is the energy of the signal $\mathbf{x}$.

Motion of waves

(!) Up to the normalized coefficient, the first basis function of the heap-transform coincides with the vector-generator.

There are three stages which can be separated during the process of motion and transformation of one basis function into another one, when starting from the wave-generator.

1− *The static stage:* the generator itself is lying as the basis function.
2− *The evolution stage* is related to the formation of a new wave.
3− *The dynamical stage,* when the new established wave is moving to the end of the path. This wave is composed by two parts. The first part resembles the generator and the second part, or a splash, is a static wave increasing by amplitude.

Example: Generator is $\mathbf{x} = (1, -1, 1, -1, 1, -1, 1, -1)'$

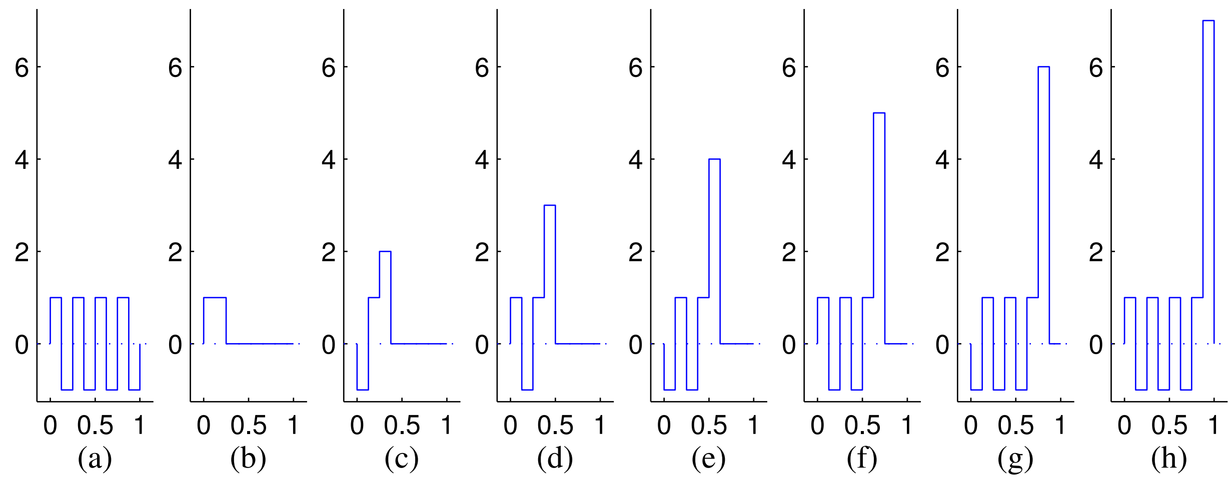


Figure 1: Non-normalized basis functions of the 8-point DsiHT generated by $\mathbf{x}$.

The generator in the last stage is trying to restore itself, while moving the large splash to the end. The maximum reconstruction of the generator is in the last movement, when it gives the much power to the splash, whose amplitude increases linearly. The generator "induces" some field and then tries to pass through it, and that is fulfilled with the loss which occurs in the form of a high but narrow splash moving to the end of the path.

Parameterized DsiHT

Let α be a positive number, and let A be a set of numbers a_k, which are defined by

$$a_k = g(x_{k-1}, x_k, \varphi_k) = \alpha^{-1} f(x_{k-1}, x_k, \varphi_k),\ \ k = 1 : N-1.$$

The basic two-point transformation, $T_\alpha(\varphi)$, in matrix form

$$T_\alpha(\varphi) \circ \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \to \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} y_0 \\ \alpha^{-1} y_0 \end{bmatrix}.$$

The angle φ and the component y_0 are calculated by

$$\tan(\varphi) = \frac{x_1 + \alpha x_0}{x_0 - \alpha x_1}, \quad y_0 = \pm \frac{\alpha}{\sqrt{1+\alpha^2}} \sqrt{x_0^2 + x_1^2}.$$

$y_0^2 = C[x_0^2 + x_1^2] < x_0^2 + x_1^2$.
Less energy is transferred to the first component but more to the second component, if $\alpha < 1$, and vice versa, if $\alpha > 1$.

Parameterized DsiHT

Example: $\alpha = 1/2$ and the generator $(x_0, x_1) = (4, 3)$.
The transform of (x_0, x_1) equals $(\sqrt{5}, 2\sqrt{5}) = (2.236, 4.472)$, the angle $\varphi = \tan^{-1}(2) = 63.4349°$.

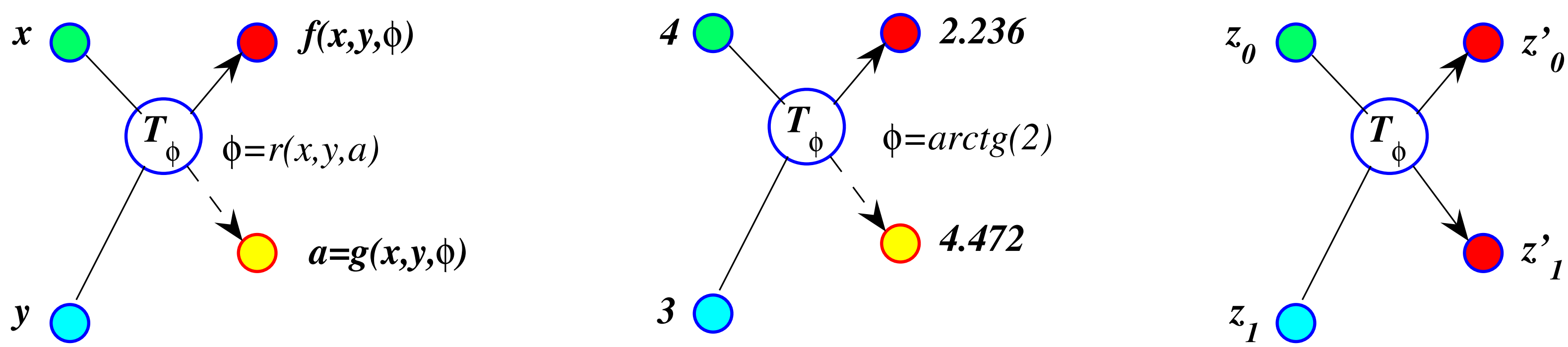


Figure 3: Graphs of the 2-point transform T_φ which is defined by the input $(x, y) = (4, 3)$ and, then, applied to a vector (z_0, z_1).

Composition of the N-point modified DsiHT

Let $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1})'$. To find values of angles φ_k, $k = 1 : (N-1)$, the decision equations are used.

<u>*Stage 1:*</u> The pair of components $\bar{x}_0 = (x_0, x_1)'$ is rotated to the vector $(y_0^{(1)}, \alpha^{-1} y_0^{(1)})'$.

<u>*Stage k :*</u> The vector $\bar{x}_{k-1} = (y_0^{(k-1)}, x_k)'$, is rotated as

$$\begin{bmatrix} y_0^{(k)} \\ \alpha^{-1} y_0^{(k)} \end{bmatrix} = \begin{bmatrix} \cos\phi_k & -\sin\phi_k \\ \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} y_0^{(k-1)} \\ x_k \end{bmatrix}, \tag{3}$$

where

$$\tan(\phi_k) = \frac{x_k + \alpha y_0^{(k-1)}}{y_0^{(k-1)} - \alpha x_k}, \tag{4}$$

$y_0^{(k-1)}$ denotes a new value of the first component on the $(k-1)$th stage, and $y_0^{(0)} = x_0$.

Composition of the N-point modified DsiHT

The first component $y_0 = y_0^{(k-1)}$ is renewed as

$$y_0^{(k)} = \pm\sqrt{Cx_k^2 + C^2 x_{k-1}^2 + \cdots + C^k x_1^2 + C^k x_0^2},$$

and the kth component equals $\alpha^{-1} y_0^{(k)}$, $k = 1:(N-1)$.
The N-point discrete transform defined by the vector $\mathbf{x}$

$$T : \mathbf{x} \to \left(y_0, \alpha^{-1} y_0^{(1)}, \alpha^{-1} y_0^{(2)}, \ldots, \alpha^{-1} y_0^{(N-1)}, \alpha^{-1} y_0 \right)$$

is called the *modified* DsiHT.

On the final stage, we obtain the following transformation of energy of the generator to the first component:

$$y_0^2 = \left(y_0^{(N-1)} \right)^2 = C^{N-1} x_0^2 + \sum_{k=1}^{N-1} C^{N-k} x_k^2 . \qquad (5)$$

Example 3: MDsiHT

Consider the case when energies of components of the signal are transferred with factors that increase as the distance of components from the original increases.
The basic transformation $T_\lambda(\phi)$ is defined as

$$\begin{bmatrix} \lambda\cos\phi & -\sin\phi \\ \lambda\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} y_0 \\ 0 \end{bmatrix} \qquad (\lambda \in (0,1))$$

$$\tan(\phi) = -x_1/(\lambda x_0), \quad y_0^2 = \lambda^2 x_0^2 + x_1^2.$$

The N-point MDsiHT based on such transformations transfers to the first component the energy which is equal to

$$y_0^2 = x_{N-1}^2 + \lambda^2 x_{N-2}^2 + \lambda^4 x_{N-3}^2 + \cdots + \lambda^{2(N-1)} x_0^2.$$

Example 3: MDsiHT

The energy of the component x_k is added to y_0 after being multiplied by factor of $\lambda^{2(N-k-1)}$, which tends to zero for large k.

If $\lambda < 1$, the energies of the first components go out; the longer the signal, the faster they go out, and $y_0^2 \le E_N^2(\mathbf{x})$.

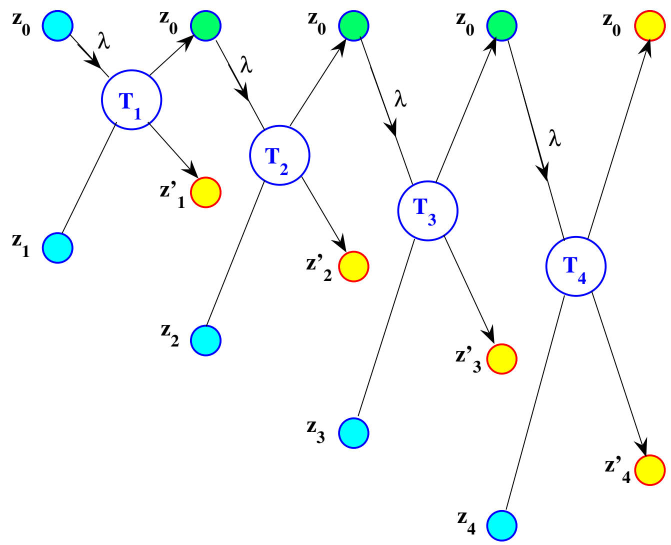


Figure 4: Signal-flow graph of calculation of the five-point MDsiHT of a vector $\mathbf{z} = (z_0, z_1, z_2, z_3, z_4)'$.

Mean angular equation

We consider the angular equations

$$a_k = g(x_{k-1}, x_k, \varphi_k) = \frac{x_{k-1} + x_k}{\lambda_k}, \qquad k = 1 : (N-1), \qquad (6)$$

when all $\lambda_k = 2$.
The basic transformation, $T(\varphi)$, is defined as

$$\begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y_0 \\ \frac{1}{2}(x+y) \end{bmatrix},$$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right) + \arccos\left(\frac{x+y}{2\sqrt{x^2+y^2}}\right),$$

and $y_0 = x\cos\varphi - y\sin\varphi$.
If we assume the conservation of energy, then

$$y_0^2 = x^2 + y^2 - \left(\frac{x+y}{2}\right)^2 > 0.$$

Examples and applications of the DsiHT

The heap transform T is composed by basic transformations

$$T_\varphi : \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \to \begin{bmatrix} y_0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}.$$

If $\mathbf{x} = (x_0, x_1, ..., x_{N-1})$ is the generator of the transform T,

$$(T[\mathbf{x}])_k = ||\mathbf{x}||\delta_k, \quad k = 0 : (N-1).$$

δ_k is the delta symbol defined as $\delta_0 = 1$ and $\delta_{k\neq 0} = 0$.
The heap transform is described in the angular form as

$$\mathbf{x} \to \mathcal{A}[\mathbf{x}] = (||\mathbf{x}||, \mathcal{A}_{N-1}[\mathbf{x}]) = (||\mathbf{x}||, \{\varphi_1, \varphi_2, ..., \varphi_{N-1}\})$$

where φ_k are the angles of rotations of basic transforms.
The first component is the energy of the signal.
Other components are angles in the interval $(-\pi, \pi)$, or $(0, 2\pi)$.

Example: signal $\mathbf{x}$ of length 256

The signal is row number 64 of the clock-and-moon image. The heap transform generated by this signal is applied to the next row of the image.

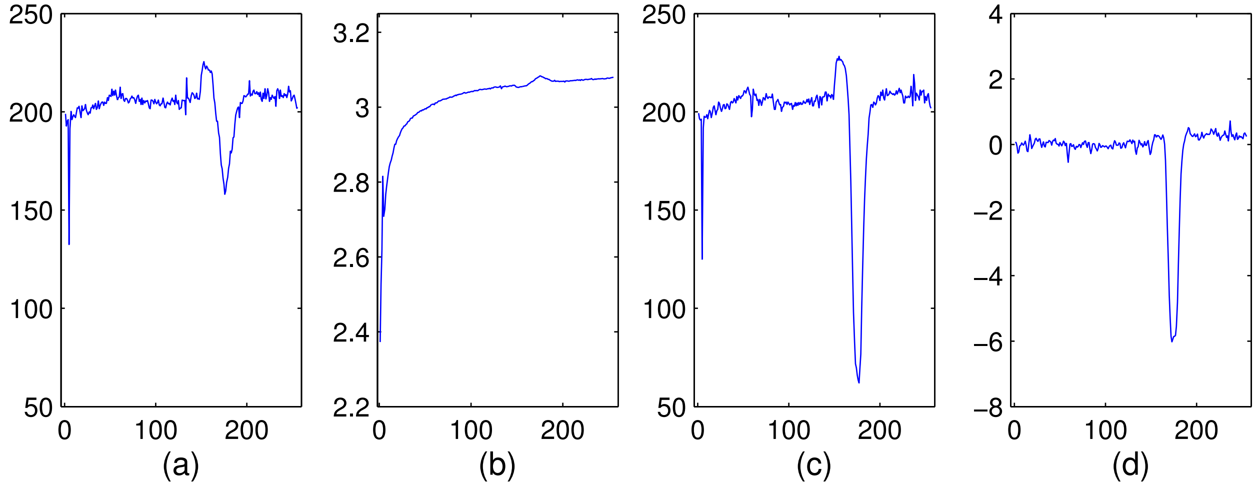


Figure 5: (a) Signal-generator, (b) the angular transform of the signal, (c) another signal, and (d) the heap transform of the signal.

The heap transform shows the interval, where the second signal differs greatly because of the noise.

Heap transforms and 2-D images

The concept of the angular transform can be used and generalized for 2-D signals and images.

For instance, we can consider the case when the DsiHTs are generated by each row of an image and then represented in their angular forms. We call such a set of 1-D heap transforms the *2D-row heap transform* and the set of corresponding angular forms the *2D-row angular transform*.

Similarly the concepts of the *2D-column heap* and *angular* transforms are defined.

Heap transforms and 2-D images

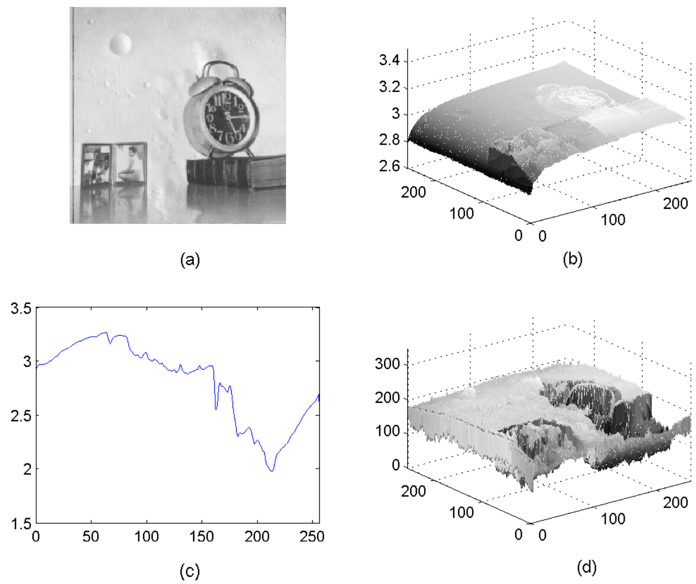


Figure 6: (a) The image, (b) mesh of the 2D-row angular transform of the image, (c) energy curve of all row-signals (normalized by 256), and (d) mesh of the image.

The picture in b resembles the light field of the angular transform with a 3-D copy of the original image on the surface of that field. Many details of the image can be observed on such a surface.

Heap transforms and 2-D images

Example of application: The 1-D heap transforms are sequentially generated by rows and applied to the next ones.

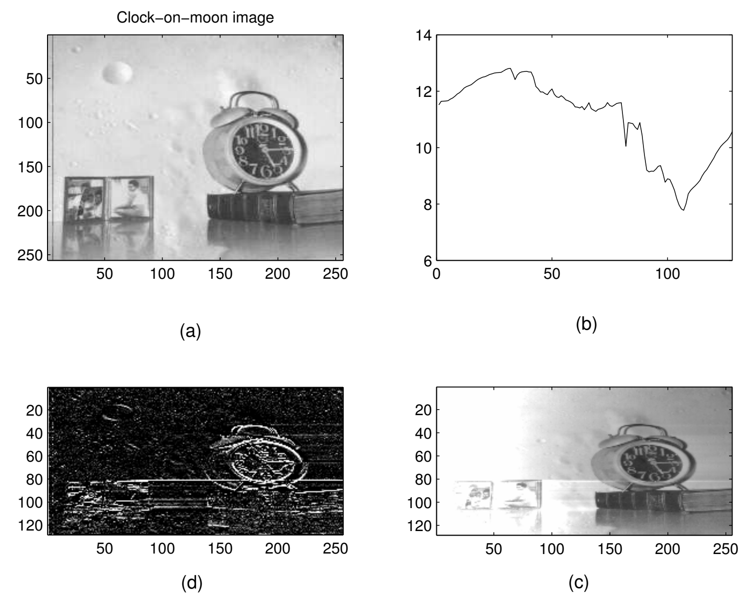


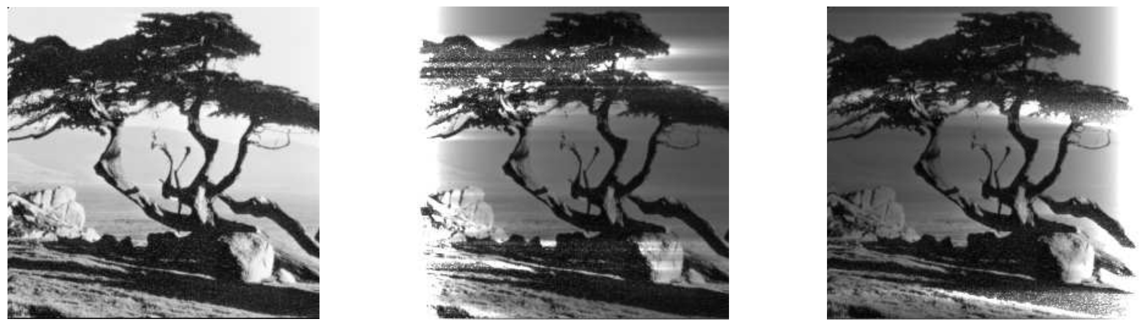
Figure 7: (a) The image, (b) energy curve of all even row-signals, (c) image of the angular transforms generated by odd-row signals, and (d) the image of the 1-D DsiHTs applied on even-row signals.

Image enhancement and angular form

Our preliminary results show that many details can be observed better through angular forms defined by different paths P, and desired properties can be obtained.

Figure 8: (a) Girl image, the 2D-row ATs with the (b) left-to-right and (c) right-to-left directions, (d) average of images in b and c, (e) 2D-column ATs with the top-to-bottom direction, and (f) average of images in b, c, and e.

Image enhancement and angular form



(a) (b) (c)

Figure 9: (a) Image, 2D-row ATs with the (b) right-to-left and (c) left-to-right directions (A part of the ground in shadow in front of the big stone becomes more visible.)

This example shows that, by performing the angular transformations along specified directions, we are able to enhance images along the directions.

We can also apply the concepts of the 2D-column and 2d-row heap transforms for processing the image block-wise, for instance by blocks 8×8 or 16×16.

Inverse angular transform (IAT)

Every 1-D signal $\mathbf{x} = \{x_0, x_1, ..., x_{N-1}\}$ can be considered in the angular form of the heap transform generated by $\mathbf{x}$.
(!) This signal itself can be considered as the angular transform, or representation of an $(N+1)$-point signal,

$$\mathbf{y} = \{y_0, y_1, ..., y_N\} \to \mathcal{A}[\mathbf{y}] = \{||\mathbf{y}||, \mathcal{A}_N[\mathbf{y}]\} = \{||\mathbf{y}||, \mathbf{x}\},$$

$$x_0 = \phi_1, x_1 = \phi_2, ..., x_{N-1} = \phi_N.$$

ϕ_k, $k = 1 : N$, are rotation angles of the heap transform generated by signal $\mathbf{y}$. Thus, we define the IAT by

$$\mathbf{x} \to \mathcal{A}_N^{-1}[\mathbf{x}].$$

To avoid ambiguity of the IAT, we assume that $||\mathbf{y}|| = 1$, i.e.

$$\mathbf{x} \to \{1, \mathbf{x}\} = \left\{1, \frac{x_0}{||x||}, \frac{x_1}{||x||}, \ldots, \frac{x_{N-1}}{||x||}\right\}.$$

Image enhancement by IAT

We can manipulate, values of the IATs along rows or columns and then perform the reconstruction of row- or column-signals.

Example: Linear amplification of the IATs of row-signals

$$\mathbf{y} = \{y_n\} \rightarrow \{(1+\alpha n)y_n\}, \ n = 0:255, \quad (\alpha = 0.005),$$

is used and, then, the angular transforms are applied.

(a) (b) (c) (d)

Figure 11: (a) Girl image, (b) 2D-row left-to-right, (c) 2D-row right-to-left, and (d) 2D-column top-to-bottom IATs of the image.

Conclusion

Examples of composing parameterized discrete unitary transforms generated by signals have been described.

The composition of such transforms is based on solving decision equations which determine the angles of rotations of basic transformations which are then used to transform the input signals.

Parameters were introduced in order to collect and transfer the energy of signals in different ways to one of the components of the generator-signals.

The application of the proposed transforms in the angular forms for image enhancement has been described.

References

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