Method of Image Enhancement by Splitting-Signals

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– EE Dept. UTSA 2005 –

Outline

- Introduction
- Transform-Based Image Enhancement and Quantative Measure of Enhancement
- Tensor Representations of Images
- Tensor Image Enhancement Techniques, or Image Enxancement by Splitting-Signals
- Conclusions

Introduction

Traditional transform-based methods of image enhancement are based on calculation of the 2-D unitary transform, for instance the discrete Fourier transform (DFT), of the image, modification of spectral components, and then calculation of the inverse transform.

The 2-D DFT can be split by different subsets of frequency-points by separate 1-D DFTs, and the problem of 2-D image processing can thus be reduced to processing separately spectral components at these subsets.

We consider the splitting which is called the *tensor representation*, and its modification, the so-called *paired representation*, which reduces the 2-D DFT to a minimal number of short 1-D DFTs.

• In this paper, a representation of an image in the form of the certain totality of 1-D "independent" splitting-signals is discussed and applied for image enhancement.

• Rather than process the image by traditional methods of the Fourier transform, we will process separately splitting-signals and then compose the 2-D DFT of the processed image, by new splitting-signals.

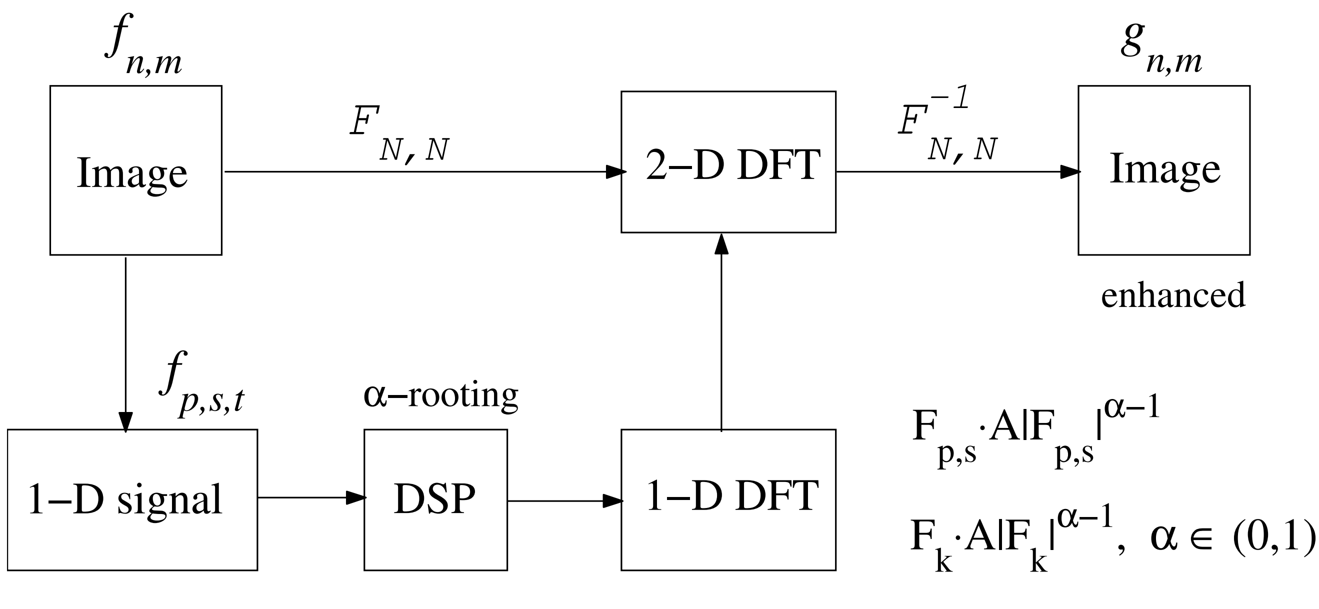


Fig. 1: Image processing by one splitting-signal.

- To estimate the quality of processed images, we consider the quantitative measure EME of image enhancement, that relates to concepts of the Weber's and Fechner's laws of the human visual system.

- Experimental results with different types of images, including aerial and medical images, show that a high quality enhancement can be achieved by processing only a few splitting-signals, and many arithmetic operations can be saved.

Splitting-Signals: The $N \times N$-point DFT of an image $f_{n,m}$ is defined by

$$F_{p,s} = (\mathcal{F}_{N,N} \circ f)_{p,s} = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f_{n,m} W^{np+ms}$$

where $W = \exp(-2\pi j/N)$.
Frequency-points (p, s) are from the set

$$X = \{(p, s);\ p, s = 0 : (N-1)\}.$$

Set X can be covered by a family of subsets $\sigma = (T_k)_{k=1:l}$, in a way that the 2-D DFT of $f_{n,m}$ at a subset T_k becomes an image of the 1-D N-point DFT of an 1-D signal, $f^{(k)}$.

A. 1-D transforms $\mathcal{F}_N$ of $f^{(k)}$ compose a splitting of the 2-D DFT

$$\mathcal{F}_{N,N}[f] \leftrightarrow \left\{\mathcal{F}_N[f^{(1)}], \mathcal{F}_N[f^{(2)}], \ldots, \mathcal{F}_N[f^{(l)}]\right\}$$

B. The set of splitting-signals $f^{(k)}$ define completely the image, $\{f^{(1)}, f^{(2)}, \ldots, f^{(l)}\} \leftrightarrow f$.

Image Tensor Representation

The covering $\sigma = (T)$ is defined by the following groups with generators (p, s)

$$T_{p,s} = \left\{(0,0), (p,s), \ldots, (\overline{(N-1)p}, \overline{(N-1)s})\right\}$$

where $\overline{p} = p(\mathrm{mod} N)$ and $T_{0,0} = \{(0,0)\}$.

Points of $T_{p,s}$ lie on parallel lines at an angle of $\theta = \tan^{-1}(p/s)$ to the horizontal axis.

The irreducible covering σ is unique. The 3×3 set X is defined by $\sigma = (T_{1,1}, T_{0,1}, T_{2,1}, T_{1,0})$.

The following property holds for the 2-D DFT

$$F_{\overline{kp},\overline{ks}} = \sum_{t=0}^{N-1} f_{p,s,t} W^{kt}, \quad k = 0\colon (N-1), \quad (1)$$

$$f_{p,s,t} = \sum_{V_{p,s,t}} f_{n,m}, \quad t = 0\colon (N-1). \quad (2)$$

Sets $V_{p,s,t} = \{(n,m);\ np + ms = t \bmod N\}$.

$V_{p,s,t}$ is the set of points (n,m) along a maximum of $p+s$ parallel straight lines defined by

$$\left.\begin{array}{rcl} xp+ys & = & t \\ xp+ys & = & t+N \\ \cdots & . & \cdots \\ xp+ys & = & t+(p+s-1)N. \end{array}\right\} \qquad (3)$$

In the bounded domain $[0,N]\times[0,N]$, these parallel lines lie at angle $\psi=\tan^{-1}(s/p)$ to the horizontal axis.

Splitting-signals (image-signals) are defined by

$$f_T = f_{T_{p,s}} = \{f_{p,s,0}, f_{p,s,1}, \ldots, f_{p,s,N-1}\}. \qquad (4)$$

$$(\mathcal{F}_{N,N}\circ f)_{|T} = \mathcal{F}_N \circ f_T\,. \qquad (5)$$

Splitting-signal $f_{T_{p,s}}$ determines the 2-D DFT at frequency-points of the group $T_{p,s}$.

The totality $\{f_T;\, T\in\sigma\}$ is called a *tensor representation* of $f_{n,m}$ with respect to the DFT. Transformation $\chi: f\rightarrow\{f_T;\, T\in\sigma\}$ is called a *tensor* transformation.

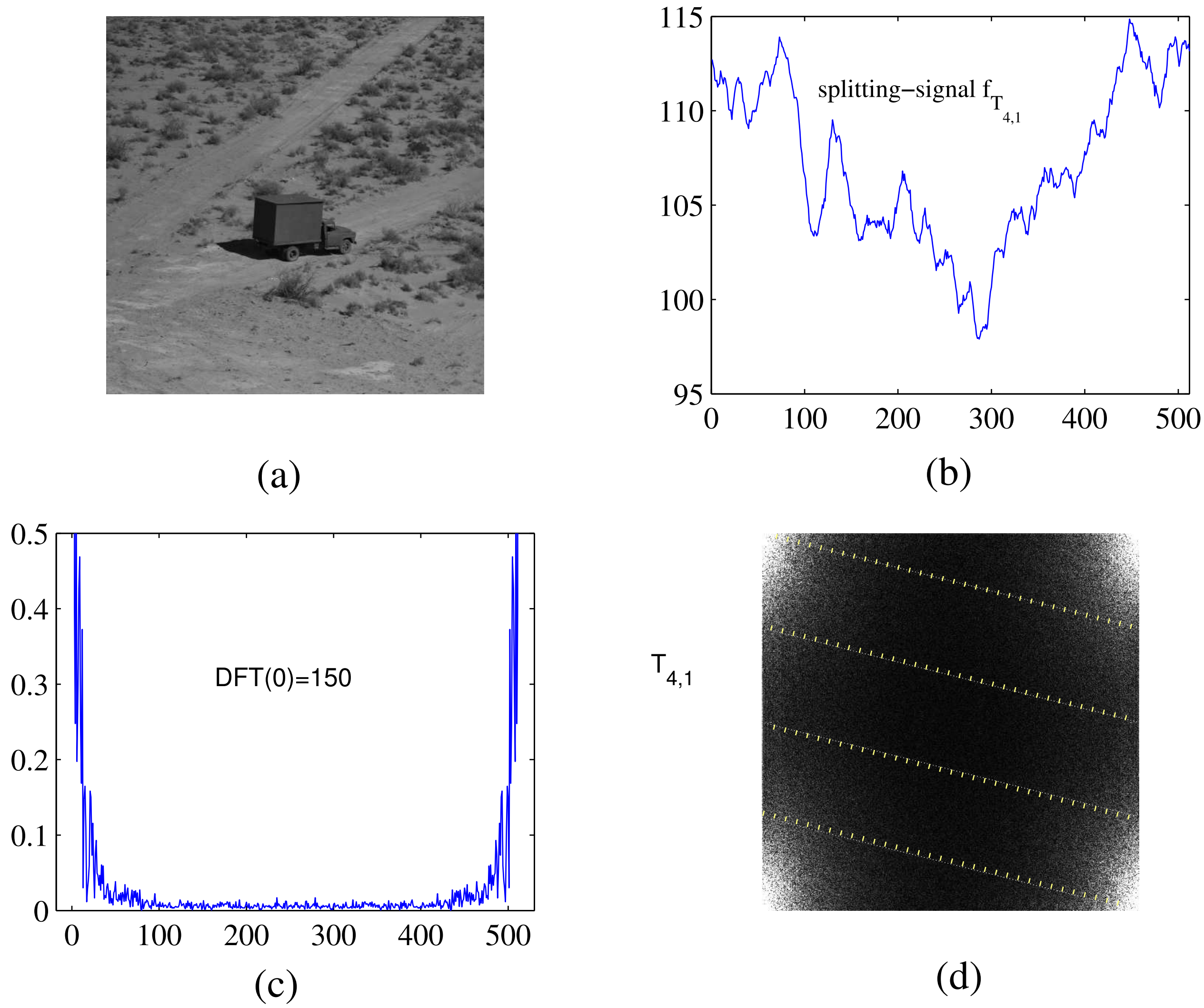


Fig. 2: (a) Truck image. (b) Splitting-signal $f_{T_{4,1}}$. (c) 1-D DFT (in absolute scale) of the splitting-signal. (d) Arrangement of values of the 1-D DFT in the 2-D DFT of the image at frequency-points of set $T_{4,1}$.

Processing of splitting-signal f_T yields a change in the Fourier spectrum at frequency-points of the corresponding group T. After performing the inverse 2-D DFT, such a change may be observed in the spatial domain at points along parallel lines of sets $V_{p,s,t}$, $t = 0 : (N-1)$.

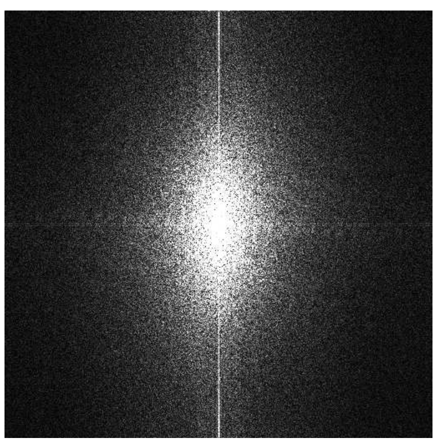
Example 1: Image after processing only one splitting-signal, (a) $f_{T_{1,6}}$ and (b) $f_{T_{1,2}}$. For illustration of directions of parallel lines of sets $V_{1,6,t}$ and $V_{1,2,t}$ on the image, magnitude of the splitting-signals were amplified by six times.

image filtered by IS−(1,6) | image filtered by IS−(1,2)

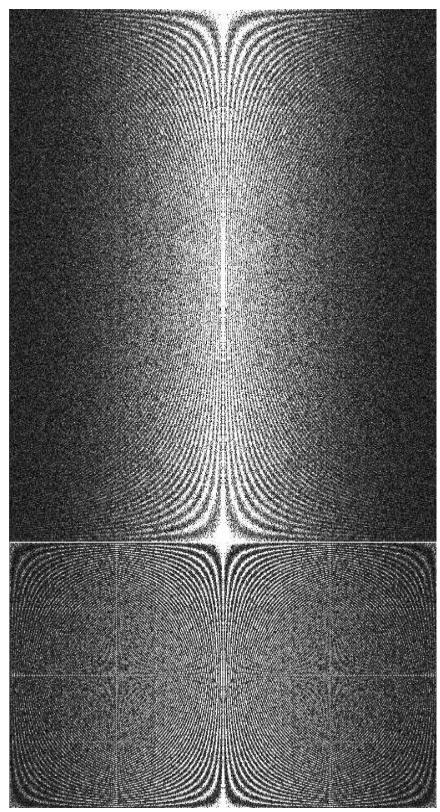
(a) | (b)

<u>2-D DFT of the Image ($N \times N$)</u>

An image is considered as the image of $3N/2$ splitting-signals and the 2-D DFT of the image as the set of $3N/2$ 1-D DFTs, when N is a power of two.



(a)



(b)

Fig. 4:(a) 2-D DFT (in absolute scale) of the truck image 512×512 and (b) image of 512-point 1-D DFTs of 768 splitting-signals.

We can select splitting-signals $f_{T_{p,s}}$ by maximums of the energy they carry,

$$E_{p,s} = \frac{1}{N} \sum_{t=0}^{N-1} f_{p,s,t}^2 = \sum_{t=0}^{N-1} \mid F_{\overline{kp},\overline{ks}} \mid^2 . \qquad (6)$$

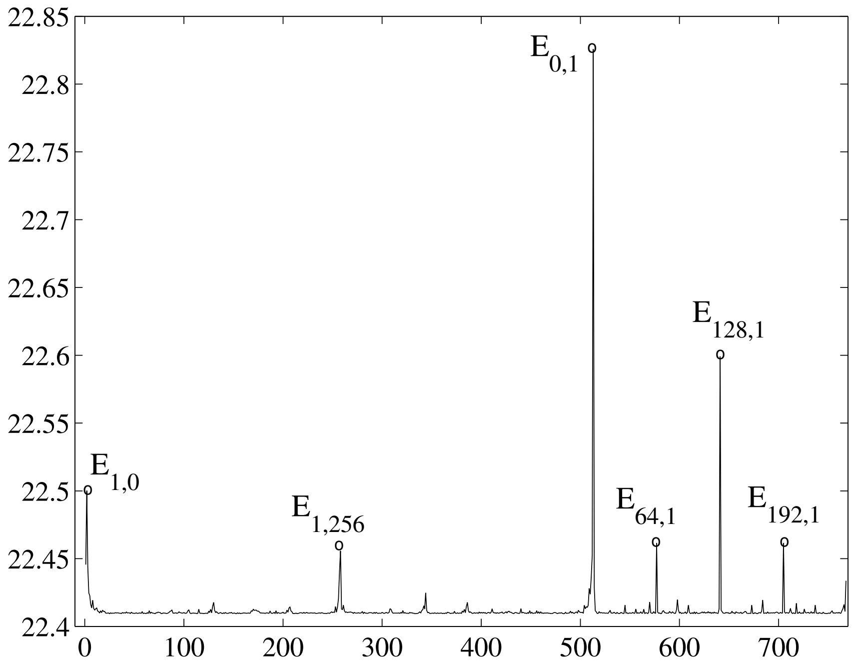


Fig. 5: The energy curve of 768 splitting-signals of the truck image.

Generators (p, s) are taken from the following covering of $X_{N,N} = \{(p, s); p, s = 0 : (N-1)\}$

$$\{\{T_{1,s}; s = 0 : N-1\}, \{T_{2p,1}; p = 0 : (N/2-1)\}\}$$

Enhancement Measure

To select optimal parameters and to measure the quality of images, the quantative measure of enhancement that relates to Weber's law of human visual system is considered.

Let g be the image obtained after processing image f by the Fourier transform-based enhancement algorithm with parameter α. A discrete image g of size $N \times N$ is divided by k^2 blocks $(L \times L)$, where $L = N/k$. The quantative measure is calculated by

$$EME_\alpha(g) = \frac{1}{k^2} \sum_{m=1}^{k} \sum_{n=1}^{k} 20 \log \left[\frac{\max_{m,n} M_\alpha(g)}{\min_{m,n} M_\alpha(g)} \right]$$

$\max_{m,n} M_\alpha(g)$ and $\min_{m,n} M_\alpha(g)$ respectively are the maximum and minimum of image g inside the (m,n)th block.

Enhancement measure of the truck image

The Fourier-transform-based image enhancement has been parameterized by α varying in the interval $[0, 1]$. The parameter $\alpha_0 = 0.92$ corresponds to the best visual estimation of enhancement. The enhancement equals

$$EME_{0.92}(g) - EME(f) = 17.43 - 9.81 = 7.62$$

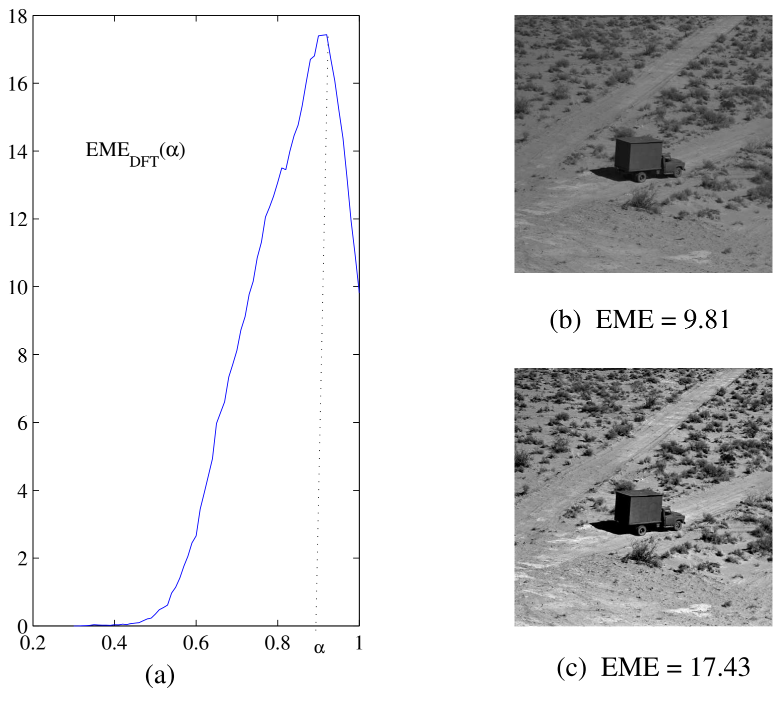


Fig. 6: Fourier transform image enhancement by the α-rooting.

Tensor Algorithm of Image Enhancement

Application of the tensor representation with the α-rooting method. Image is processed by one selected splitting-signal $f_{T_{p,s}}$.

Step 1: Perform 1-D DFT of splitting-signal

$$f_{T_{p,s}} \rightarrow F_k = \sum_{t=0}^{N-1} f_{p,s,t} W^{kt}, \quad k = 0 : (N-1).$$

Step 2: Multiply the transform of the splitting-signal by coefficients $C_k = A|F_k|^{\alpha-1}$, $k = 0 : (N-1)$, where A is a constant.

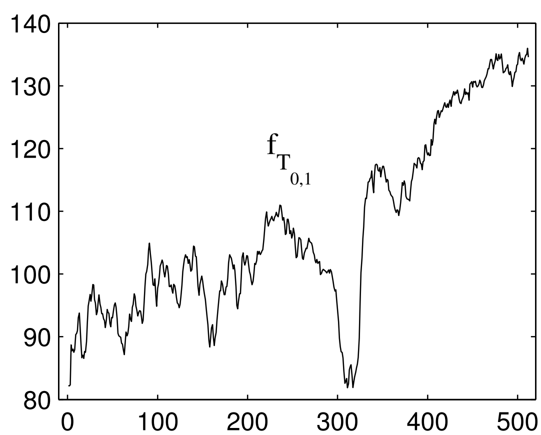
Step 3: Fill (change) the 2-D DFT by the new 1-D DFT at frequency-points of subset $T_{p,s}$.

Step 4: Perform the inverse 2-D DFT.

A: Process separately splitting-signals by different (optimal) values of parameter α.

Example: Image enhancement by the 513th splitting-signal $f_{T_{0,1}}$.

The achieved enhancement equals $EME_\alpha(g) = 16.24$, when $\alpha = 0.96$. We here recall that the traditional α-rooting by the 2-D DFT yields the optimal value 0.92 with image enhancement 17.43. The 513th splitting-signal leads to the highest enhancement by EME, when considering parameter α to be equal 0.96.



(a)

(b)

Fig. 7: (a) Splitting-signal $f_{T_{0,1}}$ and (b) image enhanced by this splitting-signal.

Enhancement by $f_{T_{1,256}}$ equals 13.45 | 0.95

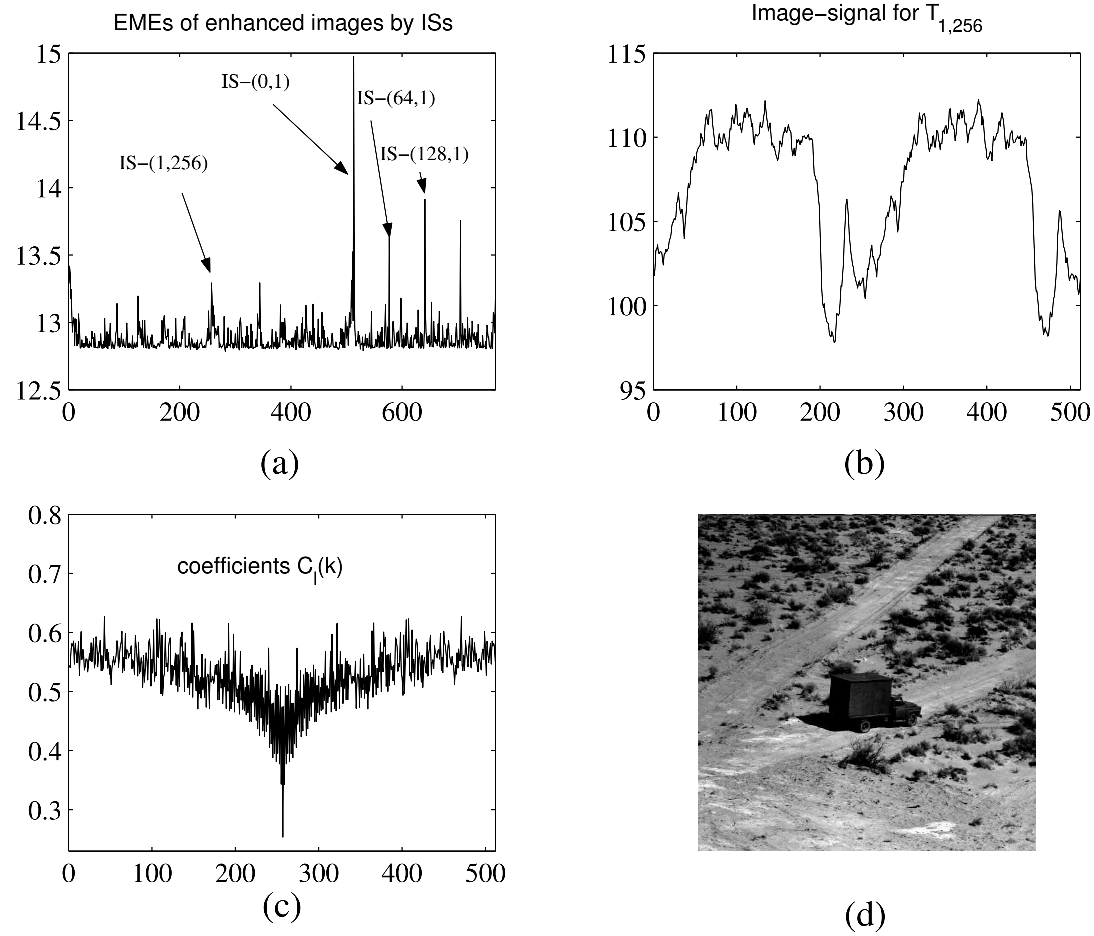


Fig. 8: (a) Enhancement measure function $EME(n, \alpha_o)$ for $\alpha_o = 0.95$, when $n = 0 : 767$. (b) splitting-signal $f_{T_{1,256}}$. (c) Coefficients $C_1(k)$ $k = 0 : 511$, of the one-dimensional α-rooting enhancement. (d) Truck image enhanced by the splitting-signal.

Enhancement by $f_{T_{1,256}}$ equals 15.94 | 0.97

The curve of $EME(256, \alpha)$, when α varies in the interval $[0.6, 1]$. The value 0.97 is optimal for this splitting-signal $f_{T_{1,256}}$ which leads to image enhancement $EME_{0.97}(g) = 15.94$.

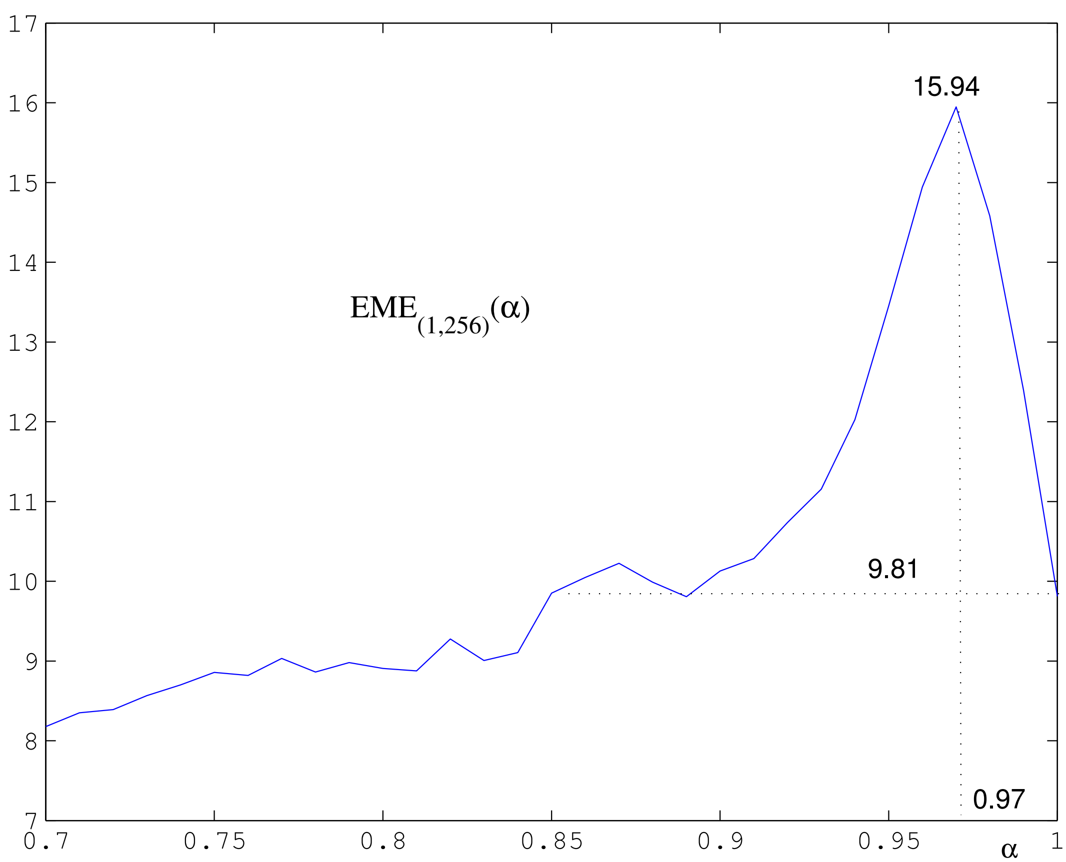


Fig. 9: Enhancement measure $EME(\alpha)$ when processing only splitting-signal # 256.

Conclution

In the new view of the image processing, the image is represented as a set of 1-D splitting-signals.

Splitting-signals split the 2-D discrete Fourier transform of the image into different groups of frequency-points.

The problem of image enhancement in the frequency domain is reduced to processing one or a few splitting-signals.

This approach allows us to achieve the enhancement and save many arithmetical operations when processing the 2-D DFT of the enhanced image.

Publications:

A.M. Grigoryan, "2-D and 1-D multi-paired transforms: Frequency-time type wavelets," *IEEE Trans. on Signal Processing*, vol. 49, no. 2, pp. 344-353, Feb. 2001.

A.M. Grigoryan and S.S. Agaian, *Multidimensional Discrete Unitary Transforms: Representation, Partitioning and Algorithms,"* Marcel Dekker Inc., New York, 2003.

A.M. Grigoryan and S.S. Agaian, "Transform-based image enhancement algorithms with performance measure," Advances in Imaging and Electron Physics, Academic Press, vol. 130, pp. 165-242, May 2004.