α-rooting Image Enhancement by Paired Splitting-Signals

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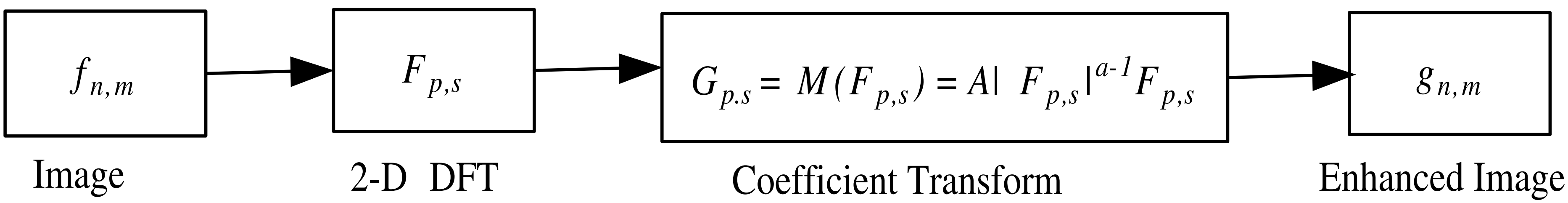
α-Rooting Technique



Quantative Measure of Enhancement

The image $g_{n,m}$ is divided by M^2 blocks $(L \times L)$, and the measure is calculated as

$$QEM_{[r]}(g) = \frac{1}{M^2} \sum_{k=1}^{M} \sum_{l=1}^{M} 20 \log_{10} \left[\frac{OR_{[r],(k,l)}(g)}{OR_{[L^2-r+1],(k,l)}(g)} \right]$$

where $OR_{[n]}(g)$ is the nth order statistic of the enhanced image g inside the (k,l)th block, when $n = r$ or $L^2 - r + 1$.

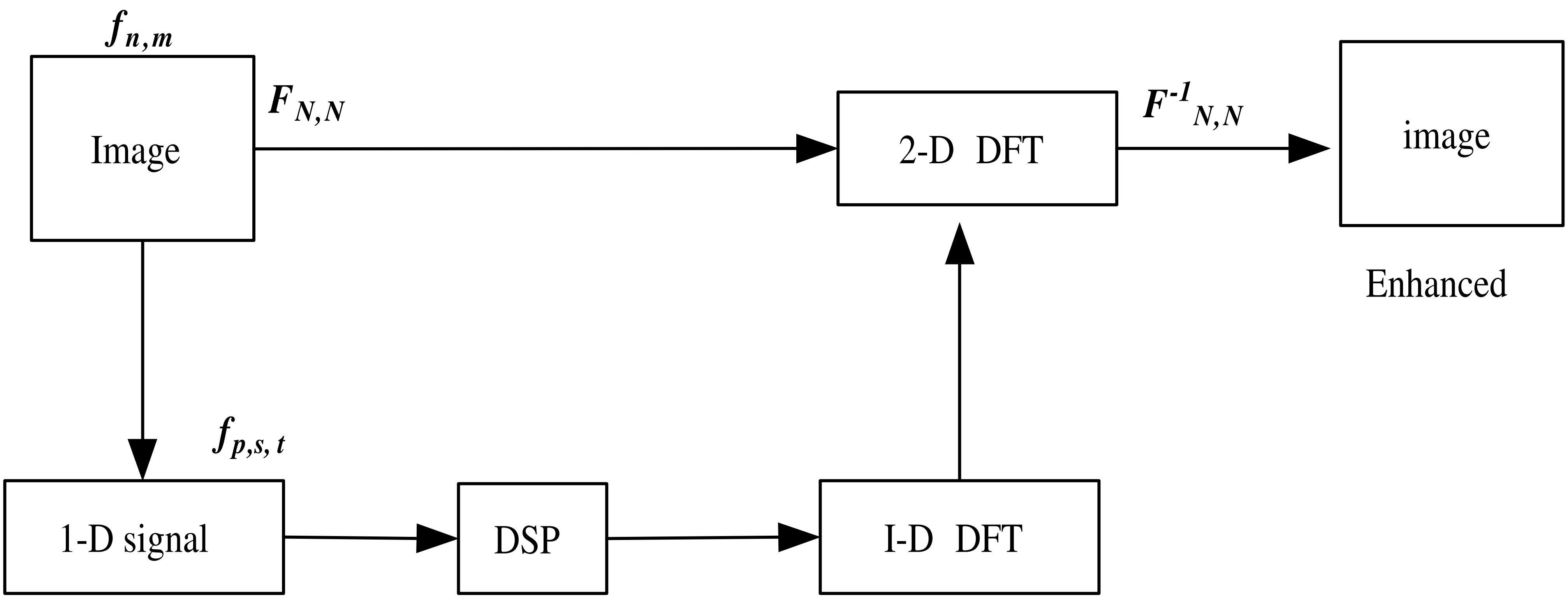
A New Look on Image and 2-D DFT

- Transfer images in forms of sets of 1-D "independent" splitting-signals.

- Tensor Representation .

- Paired Representation .

Image Processing by Tensor Transform



The block diagram image enhancement by tensor transform

Paired Representation

Paired Transform removes redundancy in Tensor Transform.

$$F_{\overline{p},\overline{s}} = \sum_{t=0}^{N-1} f'_{p,s,t} W^t, \tag{1}$$

where $f'_{p,s,t} = f_{p,s,t} - f_{p,s,t+N/2}, \quad t = 0 : (N/2 - 1)$

$$F_{\overline{(2m+1)p},\overline{(2m+1)s}} = \sum_{t=0}^{N/2-1} \left(f'_{p,s,t} W^t\right) W_{N/2}^{mt} \tag{2}$$

Paired Representation

for $m = 0 : (N/2 - 1)$. Thus the 2-D DFT of f at points of the following subset of the group $T_{p,s}$

$$T'_{p,s} = \Big\{(p,s), (\overline{3p}, \overline{3s}), , (\overline{5p}, \overline{5s}) \ldots , (\overline{(N-1)p}, \overline{(N-1)s})\Big\} \tag{3}$$

is defined by the splitting-signal of length $N/2$

$$f_{T'_{p,s}} = \{f'_{p,s,0}, f'_{p,s,1}, f'_{p,s,2}, \ldots , f'_{p,s,N/2-1}\}.$$

Image Processing by Paired Transform

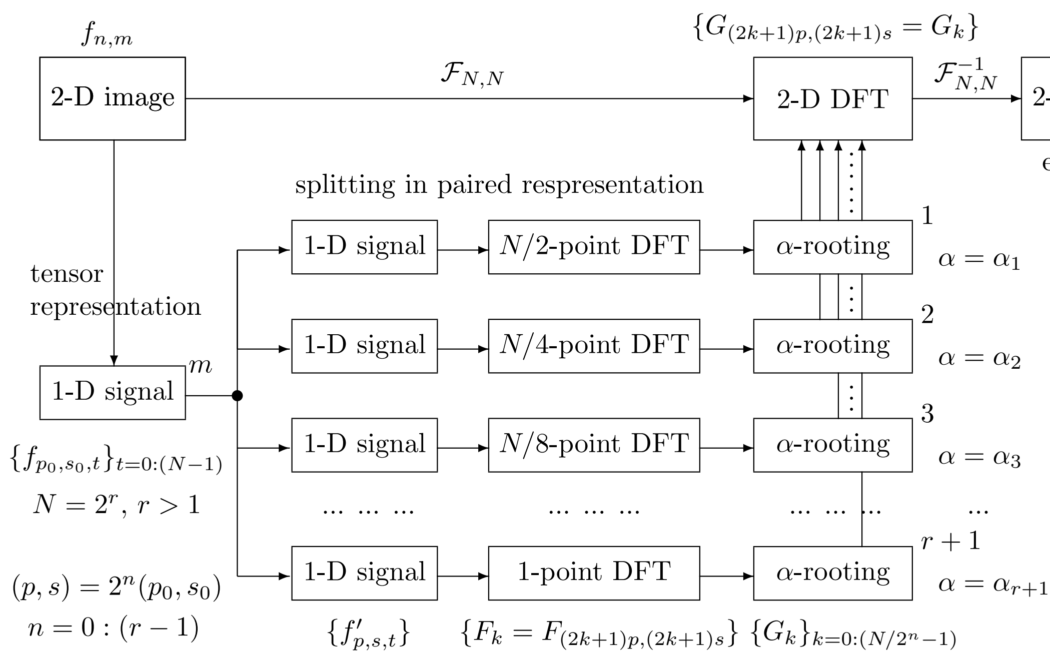
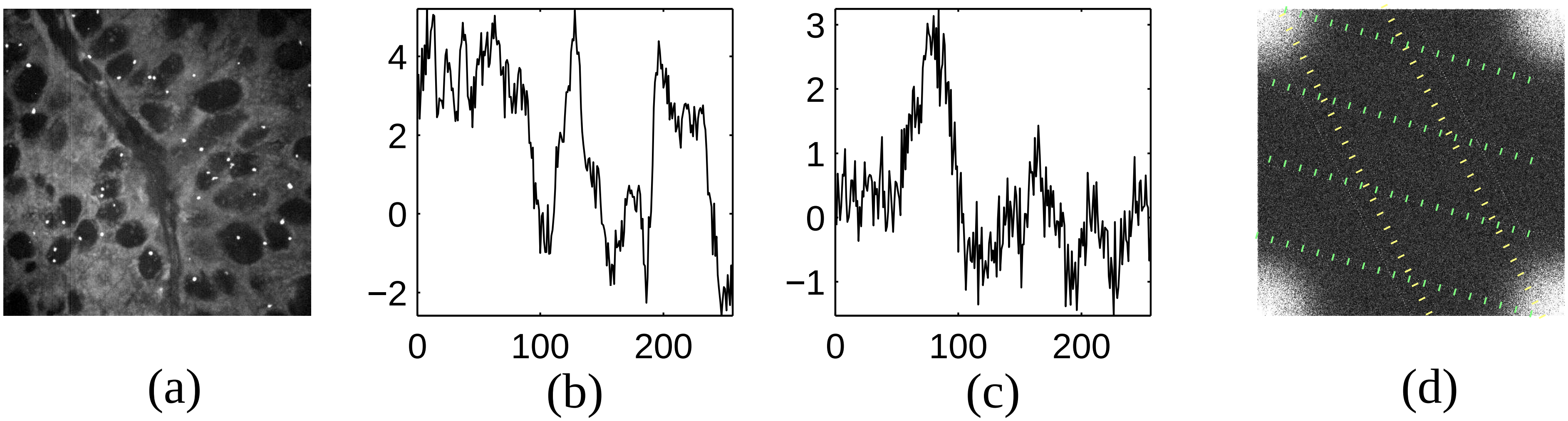


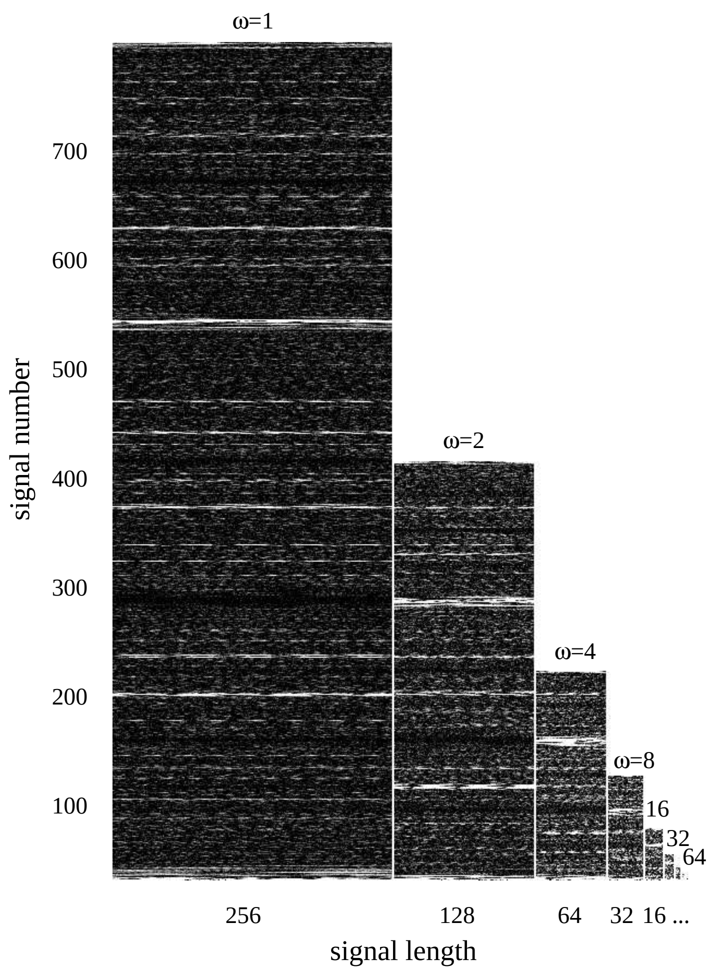
Figure 1. Image processing by splitting-signals in the paired representation.

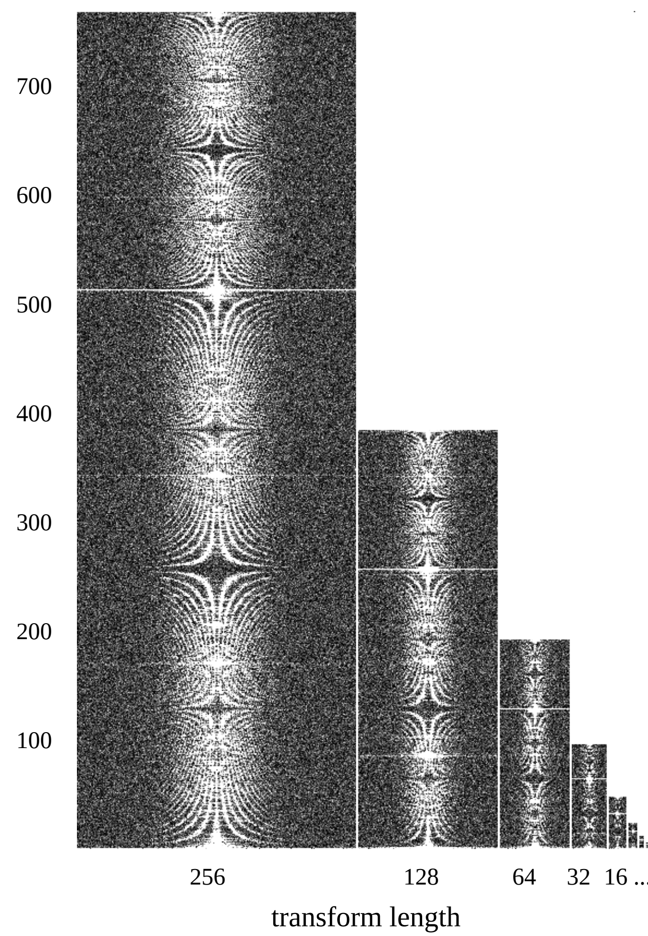
Image Processing by Paired Transform



(a) Original Image (b) Splitting-signal (c) Paired splitting-signal (d) 2-D DFT refilled by splitting-signal

Splitting-Signals of Paired Transform





(a) Splitting-signals of lengths $N/2, N/4, ..., 2, 1, 1.$ of the FISH image. (b) 1-D DFTs of the splitting-signals.

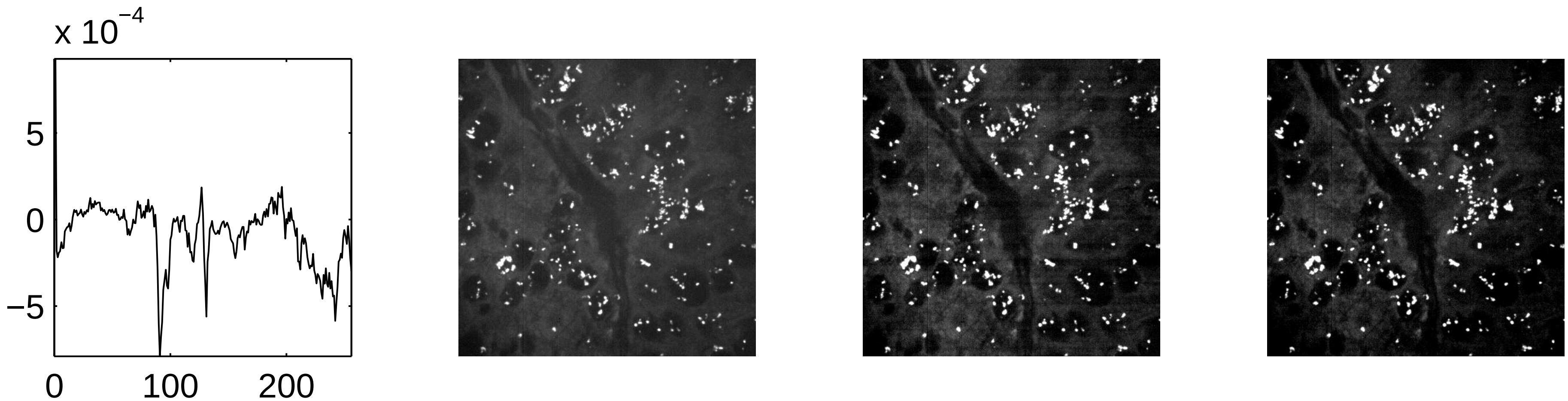
Enhancement by Fast Paired

N size coefficients C_m can be approximated to one coefficient C_0. Then the difference $(C_m - C_0)$ is defined as

$$\Delta(\alpha) = C_m - C_0 = |F_{\overline{(2m+1)p},\overline{(2m+1)s}}|^{\alpha-1} - |F_{p,s})|^{\alpha-1} \quad (4)$$

when applying the α-rooting method. It is clear, that $\Delta(\alpha) \to 0$ when α tends to 1. for $\alpha \in (0.8, 1)$.

Enhancement by Fast Paired

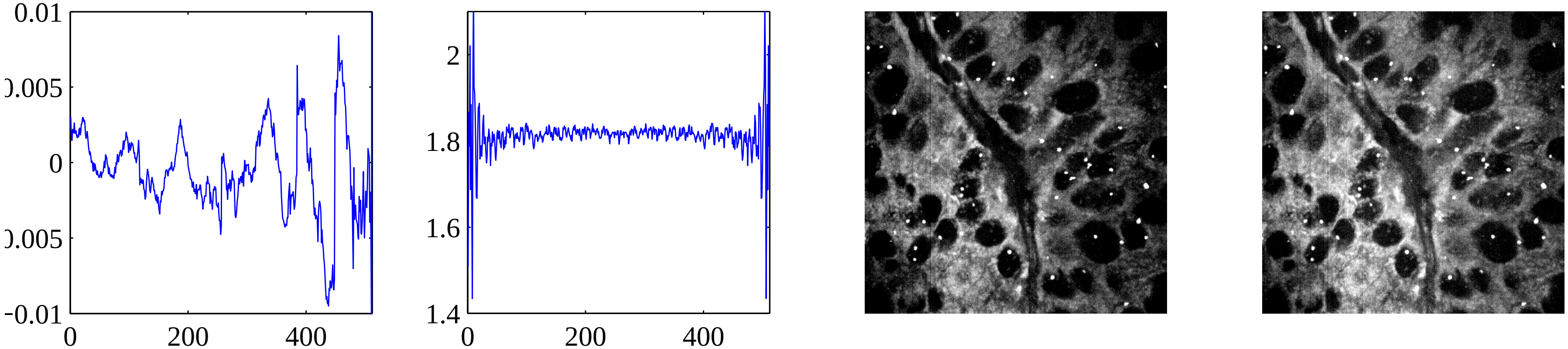


(a) (b) (c) (d)

(a) signal $e_{T'_{0,1}}$, (b) FISH image,

(c) enhancement by one coefficient, and (d) enhancement by 256 coefficients C_m.

Enhancement by Fast Paired



(a) (b) (c) (d)

(a) Signal $e_{T'_{0,1}}$, (b) 1-DFT of the signal,

(c) enhancement by one coefficient, and (d) enhancement by 256 coefficients C_k.

Splitting $\alpha-$Rooting in Spatial Domain

Enhanced image $g_{m,n}$ can be represented as:

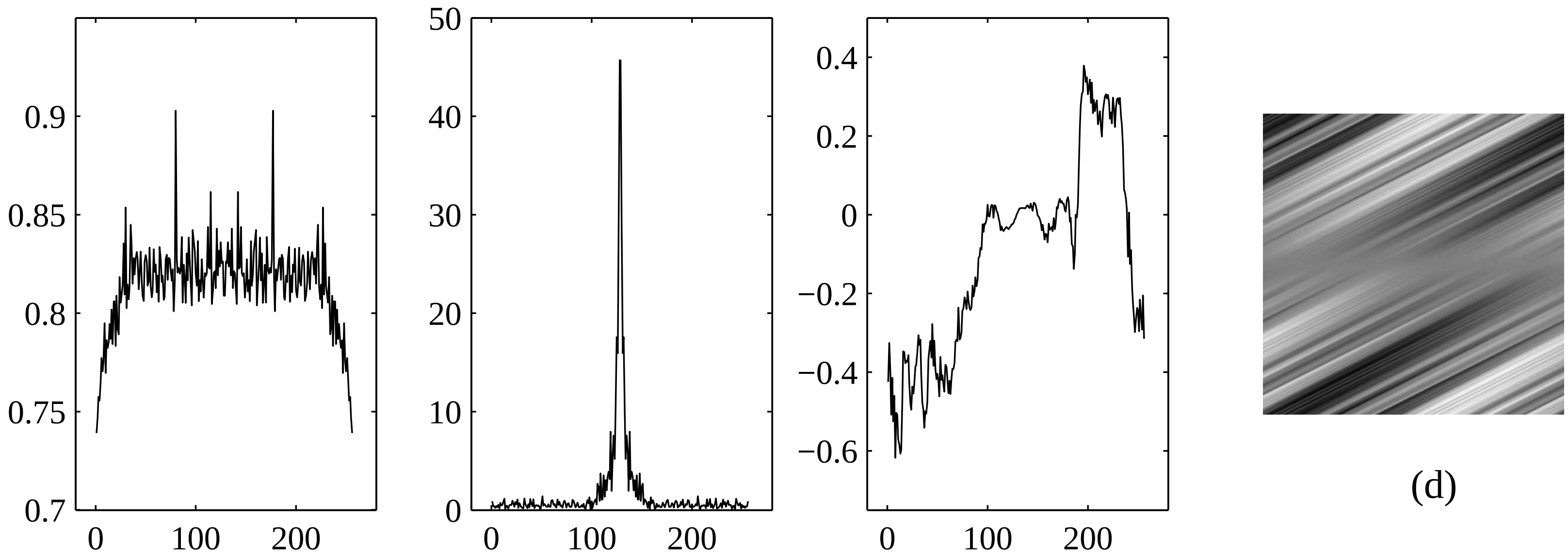
$$g_{n,m} = f_{n,m} + \frac{1}{2N}\Delta_{np+ms \bmod N/2} \cdot W^{-(np+ms)} \qquad (5)$$

where Δ_t denotes the 1-D $N/2$-point discrete signal which is the inverse N-point DFT defined by

$$\Delta_t = \frac{2}{N}\sum_{k=0}^{N/2-1}(G_k - F_k)\,W_{N/2}^{-tk},\ \ t = 0 : (N/2-1). \qquad (6)$$

Therefore, the processing of the splitting-signal $f_{T'}$ leads to the change of the image along parallel lines $np + ms = t \bmod N$, $t = 0 : (N-1)$.

Directional Effect of Splitting-signals

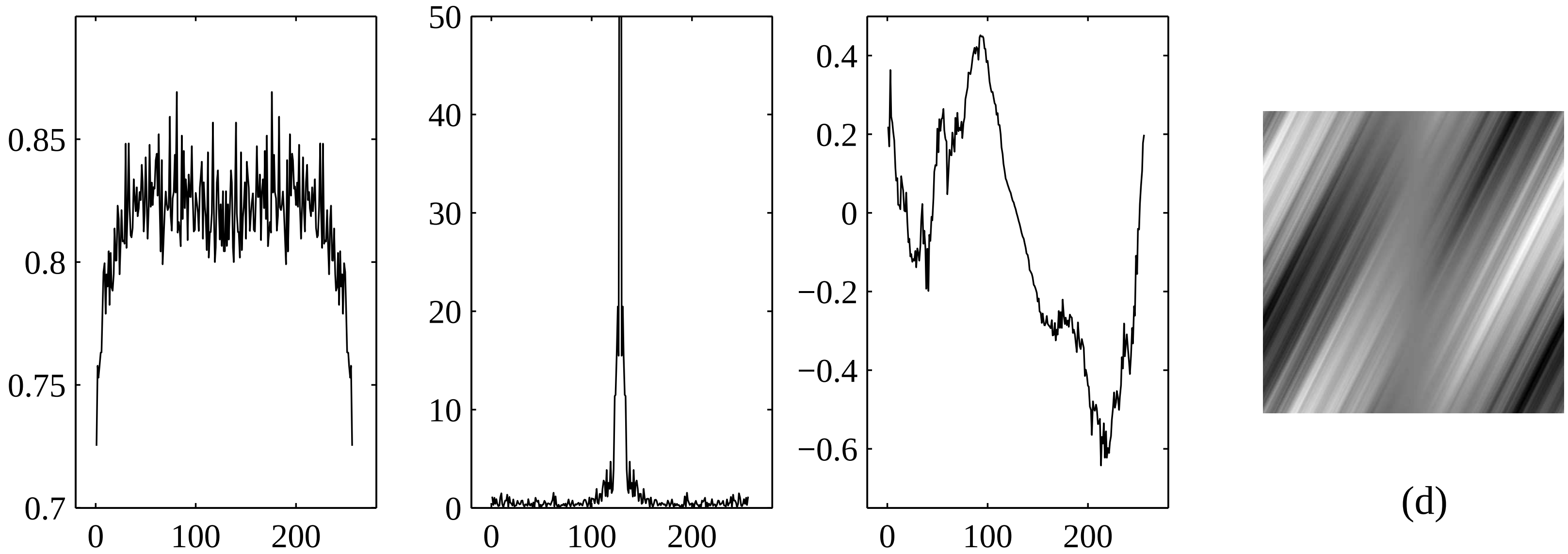


(a) (b) (c) (d)

(a) Coefficients of enhancement C_k, of the splitting-signal $f_{T'_{2,1}}$

(b) 1-D DFT of Δ_t, (c) signal Δ_t, and (d) directional image $\Delta_{n,m}$ (scaled).

Directional Effect of Splitting-signals

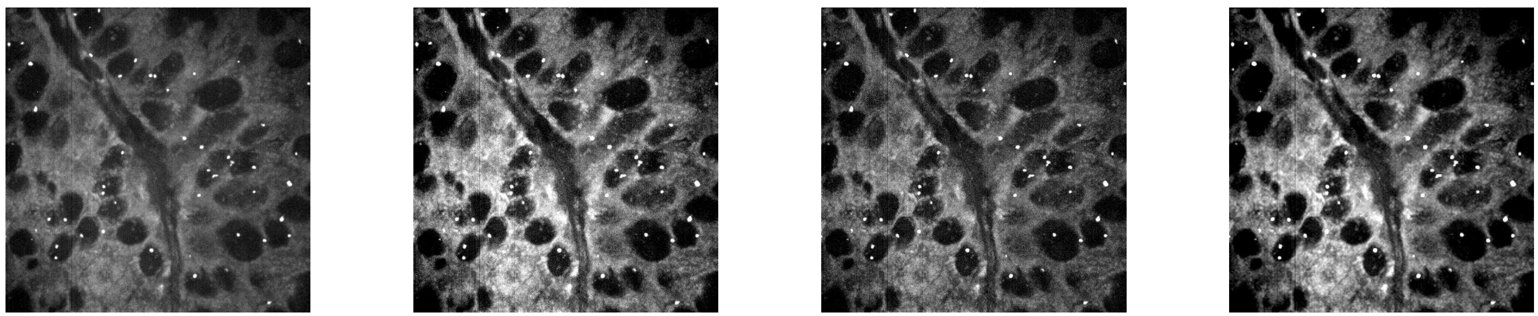


(a) (b) (c) (d)

(a) Coefficients C_k, of splitting-signal $f_{T'_{1,2}}$ of FISH image,

(b) 1-D DFT of Δ_t, (c) signal Δ_t, and (d) directional image $\Delta_{n,m}$ (scaled).

Comparison of Enhancement



(a) (b) (c) (d)

(b) splitting-signal number $(1, 0)$, (c) α-rooting, and (d) wavelet thresholding.

Conclusions

Enhancement by paired transform and its fast realization is proposed. Improved images by just processing one signal. Proposed the spatial domain application of enhancement by paired transform. Proposed methods works better than wavelet thresholding. Proposed method reached almost as good as $\alpha-$rooting with faster way.