Gradients and Compass Operators: Method of Rotations


*

Artyom M. Grigoryan[a] and Sos S. Agaian[b]

[a]Department of Electrical and Computer Engineering

The University of Texas at San Antonio, San Antonio, Texas, USA

and

[b]Computer Science Department, College of Staten Island and the Graduate Center, Staten Island, NY, USA

amgrigoryan@utsa.edu, sos.agaian@csi.cuny.edu

OUTLINE

- Introduction
- Gradients with 4 directions
- Compass gradients with 6, 8, and 16 directions
- Examples
- Comparison of gradients with 4, 8, and 16 directions
- Summary
- References

Abstract

We describe a new model of gradient operators that are defined along 16 different directions. For that, the model of two rotations of coefficients inside the given mask is proposed. The set of such gradient operators, which is called the campus gradient is described on different examples.

The 5×5 masks are considered for such operators, as the Sobel, Prewitt, Agaian gradients along 16 directions. The Nevatia-Babu, and Art-Sobel campus gradients with illustrative examples are also described.

Comparison of gradient operators along 4, 8, and 16 directions is given.

The presented approach can be easier extended for large windows and even with more than 16 directions.

GRADIENTS WITH MAXIMUM 8 DIRECTIONS

We first consider the gradient operators that are defined by using eight directions, which is the number of the coefficients of the 3×3 matrix $[G_x]$ of the gradient operator along the X-axis. The center is not counted.

The rotation of any 3×3 matrix around its center is considered counter clock-wise.

Method of Matrix Rotation:

The coefficients of the rotated matrix by 45 degree are calculated as follows:

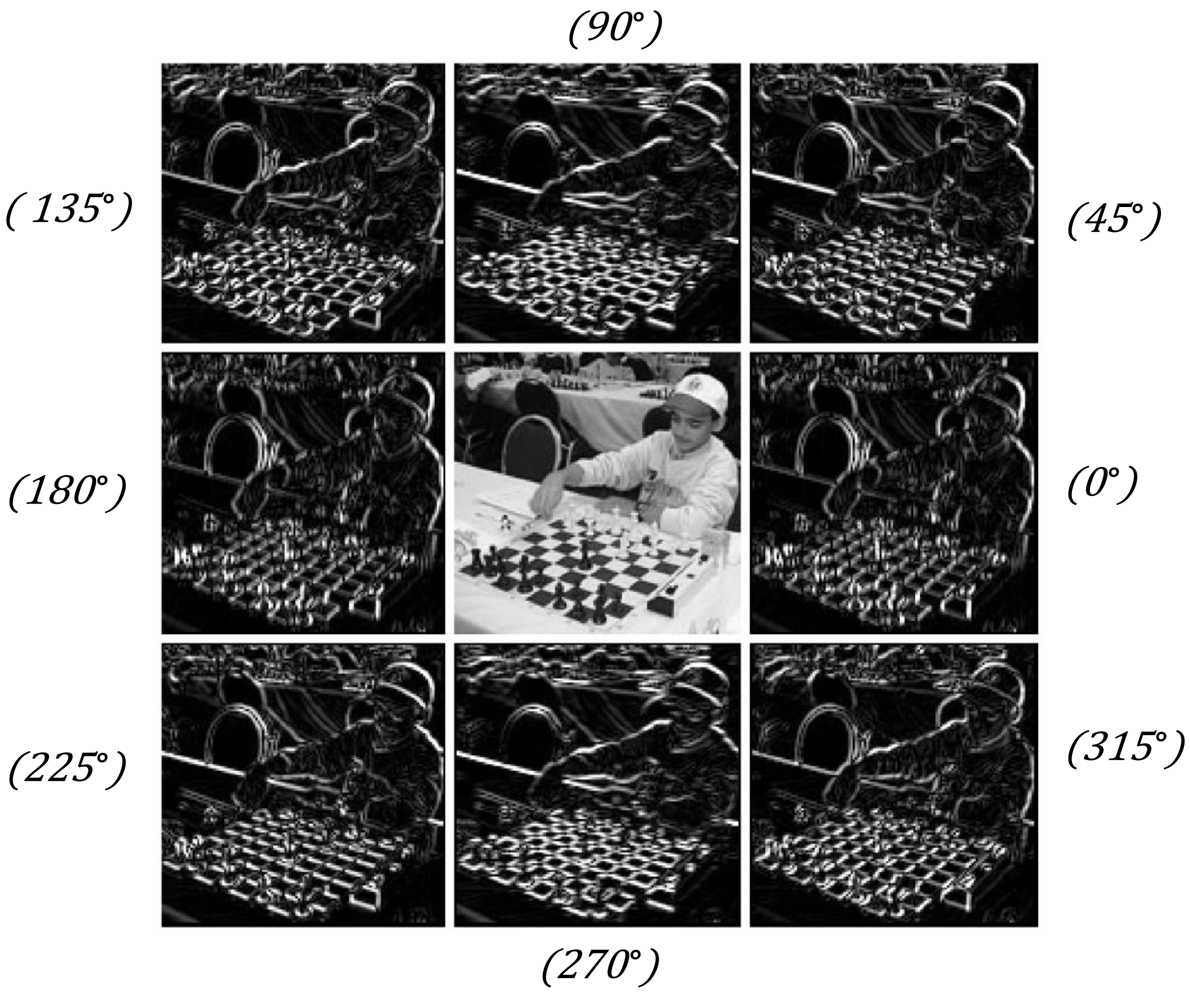
$$[G_\varphi] = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & \underline{b_2} & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \longrightarrow \begin{bmatrix} \downarrow & \leftarrow & \leftarrow \\ \downarrow & \bullet & \uparrow \\ \rightarrow & \rightarrow & \uparrow \end{bmatrix} \longrightarrow [G_{\varphi+45^\circ}] = \begin{bmatrix} a_2 & a_3 & b_3 \\ a_1 & \underline{b_2} & c_3 \\ b_1 & c_1 & c_2 \end{bmatrix}$$

For example, we consider the Kirsch gradient [2] with the following matrix:

$$[G_x^2] = G_{0^\circ} = \frac{1}{15}\begin{bmatrix} 5 & -3 & -3 \\ 5 & \underline{0} & -3 \\ 5 & -3 & -3 \end{bmatrix}. \qquad (1)$$

The scale factor 1/15 is calculated to keep the range of the image. All eight matrices of rotations by the angles φ_k of the set $\Psi = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\}$ are different, as shown below

$[G_{0^\circ}] = \frac{1}{15}\begin{bmatrix} 5 & -3 & -3 \\ 5 & \underline{0} & -3 \\ 5 & -3 & -3 \end{bmatrix}$	$[G_{45^\circ}] = \frac{1}{15}\begin{bmatrix} -3 & -3 & -3 \\ 5 & \underline{0} & -3 \\ 5 & 5 & -3 \end{bmatrix}$	$[G_{90^\circ}] = \frac{1}{15}\begin{bmatrix} -3 & -3 & -3 \\ -3 & \underline{0} & -3 \\ 5 & 5 & 5 \end{bmatrix}$
$[G_{135^\circ}] = \frac{1}{15}\begin{bmatrix} -3 & -3 & -3 \\ -3 & \underline{0} & 5 \\ -3 & 5 & 5 \end{bmatrix}$	$[G_{180^\circ}] = \frac{1}{15}\begin{bmatrix} -3 & -3 & 5 \\ -3 & \underline{0} & 5 \\ -3 & -3 & 5 \end{bmatrix}$	$[G_{225^\circ}] = \frac{1}{15}\begin{bmatrix} -3 & 5 & 5 \\ -3 & \underline{0} & 5 \\ -3 & -3 & -3 \end{bmatrix}$
$[G_{270^\circ}] = \frac{1}{15}\begin{bmatrix} 5 & 5 & 5 \\ -3 & \underline{0} & -3 \\ -3 & -3 & -3 \end{bmatrix}$	$[G_{315^\circ}] = \frac{1}{15}\begin{bmatrix} 5 & 5 & -3 \\ 5 & \underline{0} & -3 \\ -3 & -3 & -3 \end{bmatrix}$	$\sum [G_{\varphi_k}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \underline{0} & 0 \\ 0 & 0 & 0 \end{bmatrix}$



East, Northeast, North, Northwest, West, Southwest, South, and Southeast Gradients

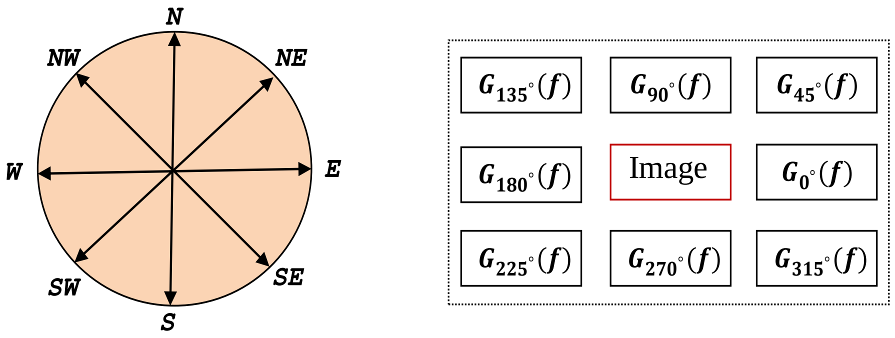


Fig 1. The grayscale image (in the middle) and Kirsch gradient images $|G_{\varphi_k}(f)|$, k=1:8.

Using the set of eight gradient images $G_{\varphi_k}(f)$, the gradient image can be calculated, as shown in the block diagram of Fig. 2.

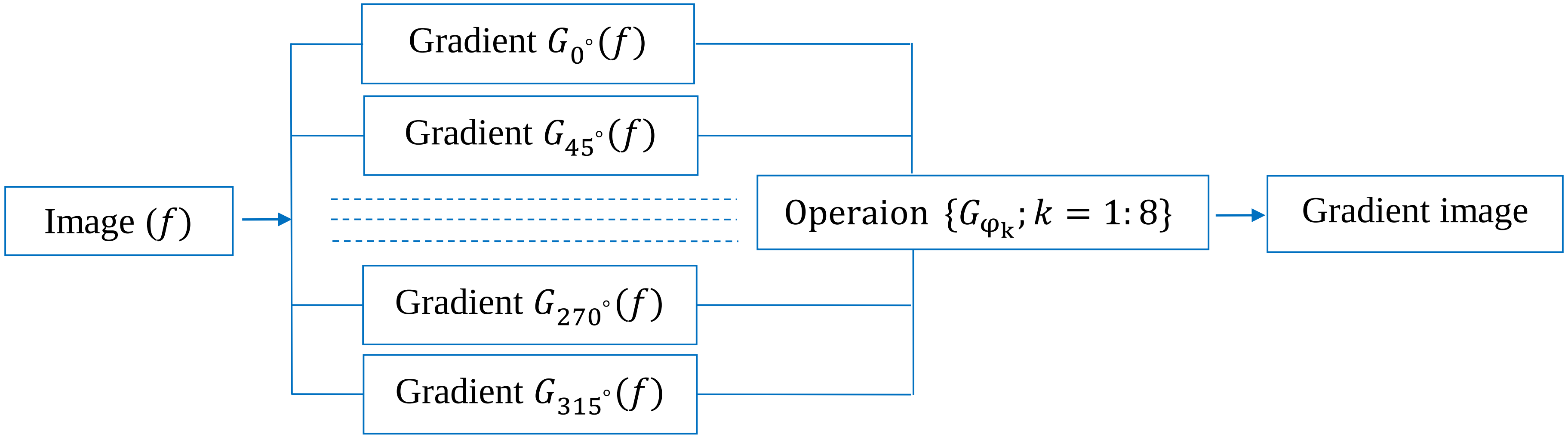


Figure 2. The diagram of calculation of the final gradient image $G_m(f)$ by eight directions.

We can define the Kirsch maximum, square-root, and magnitude gradient images

$$G_m(f) = \max\{|G_{(k-1)45^\circ}(f)|;\ k = 1{:}8\}, \qquad G^2(f) = \sqrt{\frac{1}{8}\sum_{k=1}^{8}\left[G_{(k-1)45^\circ}(f)\right]^2}\ , \qquad G(f) = \frac{1}{8}\sum_{k=1}^{8}|G_{(k-1)45^\circ}(f)|.$$

Figure 3 shows the Kirsch maximum, square-root, and magnitude gradient images composed by eight oriented gradients images in parts (a), (b), and (c), respectively.

(a) maximum (b) sum (c) square-root

Figure 3. The Kirsch (a) maximum, (b) square-root, and (d) magnitude gradient images.

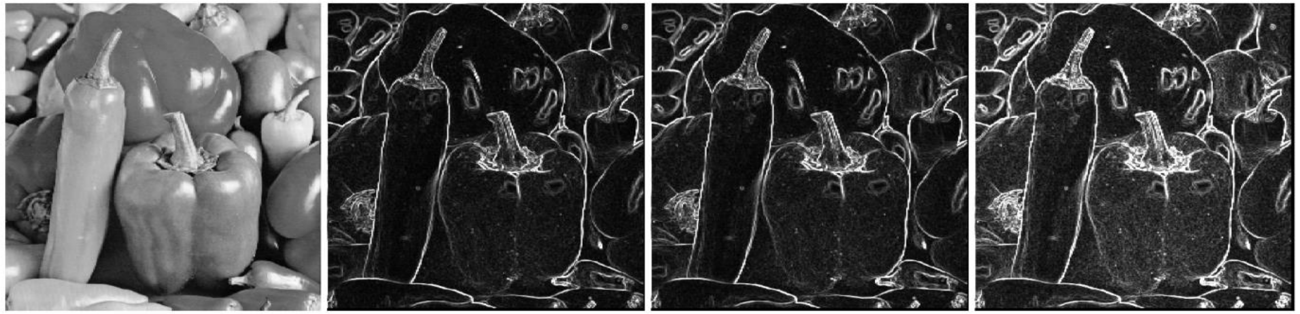


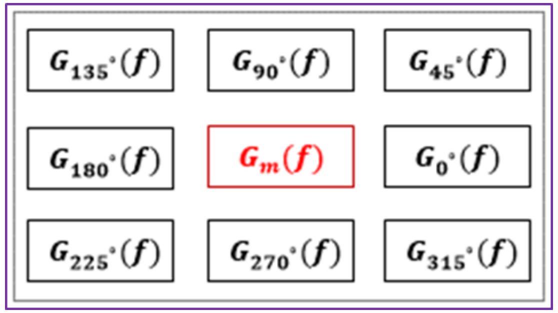
Fig. 4 (a) The “pepper” image and the gradients: (b) the magnitude, (c) the square-root, and (d) the maximum gradient images by eight directions.

The Art Gradient Operators with 8 Directions

The eight matrices of rotations by the angles φ_k of the set

$$\Psi = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\},$$

$[G_{0^\circ}] = \frac{1}{8}\begin{bmatrix} 2 & -2 & -1 \\ 4 & \underline{0} & -2 \\ 2 & -2 & -1 \end{bmatrix}$	$[G_{45^\circ}] = \frac{1}{8}\begin{bmatrix} -2 & -1 & -2 \\ 2 & \underline{0} & -1 \\ 4 & 2 & -2 \end{bmatrix}$	$[G_{90^\circ}] = \frac{1}{8}\begin{bmatrix} -1 & -2 & -1 \\ -2 & \underline{0} & -2 \\ 2 & 4 & 2 \end{bmatrix}$
$[G_{135^\circ}] = \frac{1}{8}\begin{bmatrix} -2 & -1 & -2 \\ -1 & \underline{0} & 2 \\ -2 & 2 & 4 \end{bmatrix}$	$[G_{180^\circ}] = \frac{1}{8}\begin{bmatrix} -1 & -2 & 2 \\ -2 & \underline{0} & 4 \\ -1 & -2 & 2 \end{bmatrix}$	$[G_{225^\circ}] = \frac{1}{8}\begin{bmatrix} -2 & 2 & 4 \\ -1 & \underline{0} & 2 \\ -2 & -1 & -2 \end{bmatrix}$
$[G_{270^\circ}] = \frac{1}{8}\begin{bmatrix} 2 & 4 & 2 \\ -2 & \underline{0} & -2 \\ -1 & -2 & -1 \end{bmatrix}$	$[G_{315^\circ}] = \frac{1}{8}\begin{bmatrix} 4 & 2 & -2 \\ 2 & \underline{0} & -1 \\ -2 & -1 & -2 \end{bmatrix}$	$\sum [G_{\varphi_k}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \underline{0} & 0 \\ 0 & 0 & 0 \end{bmatrix}$



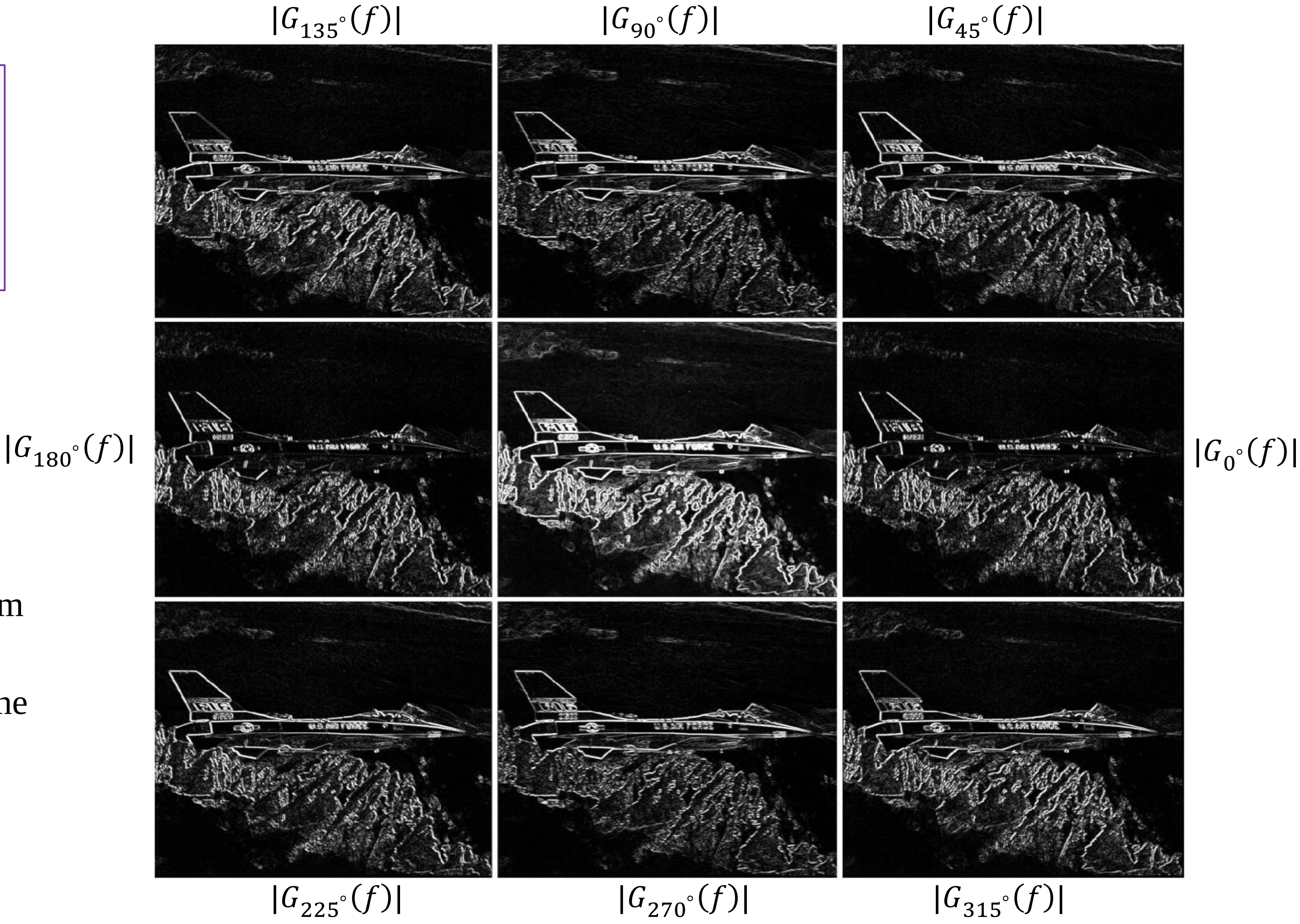


Fig. 5 The Art maximum gradient image (in the middle) together with the eight gradient images $|G_{\varphi_k}(f)|$, k=1:8.

We can define the square-root and magnitude gradient image by

$$G^2(f) = \sqrt{\frac{1}{8}\sum_{k=1}^{8}\left[G_{(\mathrm{k}-1)45^\circ}(f)\right]^2} \quad \text{and} \quad G(f) = \frac{1}{8}\sum_{k=1}^{8}\left|G_{(\mathrm{k}-1)45^\circ}(f)\right|.$$

(a) maximum (b) sum (c) square-root

Fig. 6 The Art (a) maximum, (b) magnitude, and (d) square-root gradient images.

Gradients with 16 directions

To add more directions for gradient operators, we need to use the gradient matrices of large size. For instance, for the points in the 5×5 window, there are four additional directions can be considered, as shown in part (b). Therefore, along these directions, new eight gradient operators can be defined. Together with the eight gradients in the main compass directions, the number of gradient operators is 16.

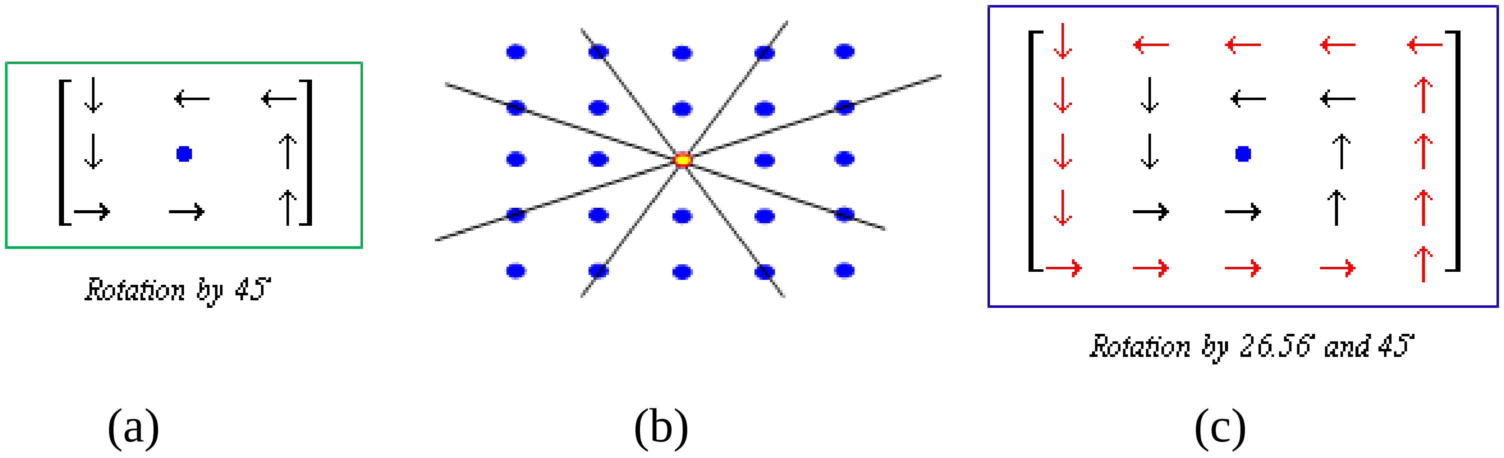
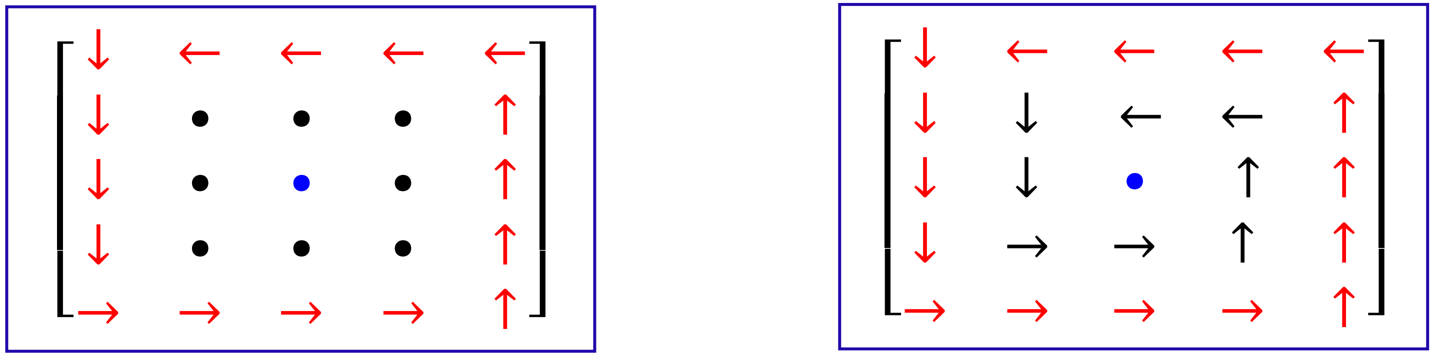


Fig. 7 (a) Counter clock-wise rotation of the 3×3 matrix, (b) the lattice 5×5, and (c) the rotation of the 5×5 matrix.

A rotation of the 5×5 gradient matrix: it is the counter clock-wise rotation of the coefficients of the matrix that is shown in the figure. The rotations can be described as follows. The 16 coefficients of the matrix in the 1st and 5th rows and columns are rotated counter clock-wise on each stage of the rotation, as shown in part (a). The other eight coefficients that are closer to the center of the matrix are rotated on each second stage of the rotation, as shown in part (b).



Rotation of only 16 coefficients

(a)

Rotation of all coefficients

(b)

Fig. 8 Counter clock-wise rotation of the matrix: (a) the 1st step and (b) the 2nd step.

Thus, the two consequent rotations are described as follows:

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \\ d_1 & d_2 & d_3 & d_4 & d_5 \\ e_1 & e_2 & e_3 & e_4 & e_5 \end{bmatrix} \rightarrow \begin{bmatrix} a_2 & a_3 & a_4 & a_5 & b_5 \\ a_1 & b_2 & b_3 & b_4 & c_5 \\ b_1 & c_2 & c_3 & c_4 & d_5 \\ c_1 & d_2 & d_3 & d_4 & e_5 \\ d_1 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} \rightarrow \begin{bmatrix} a_3 & a_4 & a_{45} & b_5 & c_5 \\ a_2 & b_3 & b_4 & c_4 & d_5 \\ a_1 & b_2 & c_3 & d_4 & e_5 \\ b_1 & c_2 & d_2 & d_3 & e_4 \\ c_1 & d_1 & e_1 & e_2 & e_3 \end{bmatrix}$$

Original matrix the 1st rotation the 2nd rotation

The eight main directions in the compass are with the angles

$$\Phi = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ\}.$$

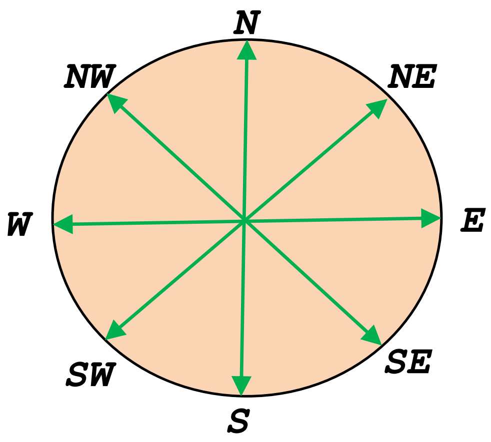
Let ψ be $\operatorname{atan}(1/2) = 26.5651^\circ \approx 26.56^\circ$. Then, we also consider the new set of 8 angles

$$\Phi_\psi = \{\psi, 90^\circ - \psi, 90^\circ + \psi, 180^\circ - \psi, 180^\circ + \psi, 270^\circ - \psi, 270^\circ + \psi, 360^\circ - \psi\}$$
$$= \{26.56°, 63.43°, 116.56°, 153.43°, 206.56°, 243.43°, 296.56°, 333.43°\},$$

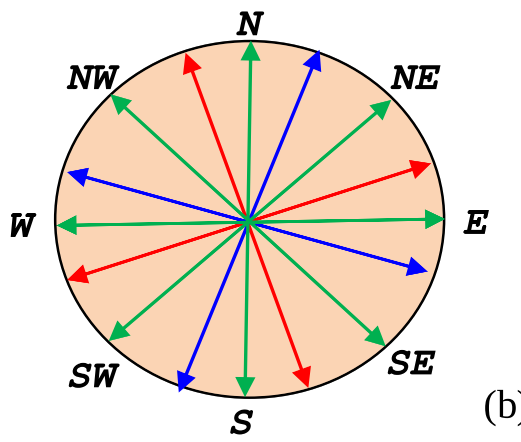
The complete set of 16 angles $\Psi = \{\Phi, \Phi_\psi\}$ of rotations can be written in the ascending order as

$$\Psi = \begin{Bmatrix} 0^\circ, 26.56°, 45^\circ, 63.43°, 90^\circ, 116.56°, 135^\circ, 153.43°, 180^\circ, \\ 206.56°, 225^\circ, 243.43°, 270^\circ, 296.56°, 315^\circ, 333.43° \end{Bmatrix}. \quad (4)$$

Figure 9 shows the eight main directions of the campus in part (a) and the additional eight directions by the angles of Φ_ψ in part (b).



(a)



(b)

Fig. 9 Eight and sixteen directions in the compass.

Denoting by $\psi_k, k = 1{:}16,$ the angles of the set Ψ, we call $[G_{\psi_k}]$ the matrix obtained by rotating the gradient matrix $[G_x]$ by the angle ψ_k. Then, the gradient image calculated by the gradient operator with the matrix $[G_{\psi_k}]$ is denoted by $G_{\psi_k}(f)$. For simplicity of notations, we round the angles to integers and consider the set Ψ as

$$\Psi = \left\{\begin{matrix} 0^\circ, 27^\circ, 45^\circ, 63^\circ, 90^\circ, 117^\circ, 135^\circ, 153^\circ, 180^\circ, \\ 207^\circ, 225^\circ, 243^\circ, 270^\circ, 297^\circ, 315^\circ, 333^\circ \end{matrix}\right\}.$$

Therefore, all 16 gradient operators, which we call *the oriented gradient operators*, can be denoted as

$$\mathrm{G}_\Psi = \left\{\begin{matrix} G_{0^\circ}, G_{27^\circ}, G_{45^\circ}, G_{63^\circ}, G_{90^\circ}, G_{117^\circ}, G_{135^\circ}, G_{153^\circ}, \mathrm{G}_{180^\circ}, \\ G_{207^\circ}, G_{225^\circ}, G_{243^\circ}, G_{270^\circ}, G_{297^\circ}, G_{315^\circ}, G_{333^\circ}. \end{matrix}\right\} \qquad (5)$$

A. The Sobel's Oriented Gradients

We consider the Sobel's differencing operator in the horizontal direction that has the matrix (without the scale factor 1/48)

$$[G_{0^\circ}] = [G_x^2] = \begin{bmatrix} 1 & 2 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & \underline{0} & -12 & -6 \\ 4 & 8 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix}. \qquad (6)$$

The first two rotations of this gradient matrix result in the following matrices:

$$\begin{bmatrix} 1 & 2 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & \underline{0} & -12 & -6 \\ 4 & 8 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & -2 & -1 & -4 \\ 1 & 8 & 0 & -8 & -6 \\ 4 & 12 & \underline{0} & -12 & -4 \\ 6 & 8 & 0 & -8 & -1 \\ 4 & 1 & 2 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -2 & -1 & -4 & -6 \\ 2 & 0 & -8 & -12 & -4 \\ 1 & 8 & \underline{0} & -8 & -1 \\ 4 & 12 & 8 & 0 & -2 \\ 6 & 4 & 1 & 2 & 0 \end{bmatrix}.$$

Continuing similar rotations, we obtain 16 matrices of the Sobel's gradient operators

$$[G_{0^\circ}] = \begin{bmatrix} 1 & 2 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & \underline{0} & -12 & -6 \\ 4 & 8 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix}$$

$$[G_{27^\circ}] = \begin{bmatrix} 2 & 0 & -2 & -1 & -4 \\ 1 & 8 & 0 & -8 & -6 \\ 4 & 12 & \underline{0} & -12 & -4 \\ 6 & 8 & 0 & -8 & -1 \\ 4 & 1 & 2 & 0 & -2 \end{bmatrix}$$

$$[G_{45^\circ}] = \begin{bmatrix} 0 & -2 & -1 & -4 & -6 \\ 2 & 0 & -8 & -12 & -4 \\ 1 & 8 & \underline{0} & -8 & -1 \\ 4 & 12 & 8 & 0 & -2 \\ 6 & 4 & 1 & 2 & 0 \end{bmatrix}$$

$$[G_{63^\circ}] = \begin{bmatrix} -2 & -1 & -4 & -6 & -4 \\ 0 & 0 & -8 & -12 & -1 \\ 2 & 8 & \underline{0} & -8 & -2 \\ 1 & 12 & 8 & 0 & 0 \\ 4 & 6 & 4 & 1 & 2 \end{bmatrix}$$

$$[G_{90^\circ}] = \begin{bmatrix} -1 & -4 & -6 & -4 & -1 \\ -2 & -8 & -12 & -8 & -2 \\ 0 & 0 & \underline{0} & 0 & 0 \\ 2 & 8 & 12 & 8 & 2 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$[G_{117^\circ}] = \begin{bmatrix} -4 & -6 & -4 & -1 & -2 \\ -1 & -8 & -12 & -8 & 0 \\ -2 & 0 & \underline{0} & 0 & 2 \\ 0 & 8 & 12 & 8 & 1 \\ 2 & 1 & 4 & 6 & 4 \end{bmatrix}$$

$$[G_{135^\circ}] = \begin{bmatrix} -6 & -4 & -1 & -2 & 0 \\ -4 & -12 & -8 & 0 & 2 \\ -1 & -8 & \underline{0} & 8 & 1 \\ -2 & 0 & 8 & 12 & 4 \\ 0 & 2 & 1 & 4 & 6 \end{bmatrix}$$

$$[G_{153^\circ}] = \begin{bmatrix} -4 & -1 & -2 & 0 & 2 \\ -6 & -12 & -8 & 0 & 1 \\ -4 & -8 & \underline{0} & 8 & 4 \\ -1 & 0 & 8 & 12 & 6 \\ -2 & 0 & 2 & 1 & 4 \end{bmatrix}$$

$$[G_{180^\circ}] = \begin{bmatrix} -1 & -2 & 0 & 2 & 1 \\ -4 & -8 & 0 & 8 & 4 \\ -6 & -12 & \underline{0} & 12 & 6 \\ -4 & -8 & 0 & 8 & 4 \\ -1 & -2 & 0 & 2 & 1 \end{bmatrix}$$

$$[G_{207^\circ}] = \begin{bmatrix} -2 & 0 & 2 & 1 & 4 \\ -1 & -8 & 0 & 8 & 6 \\ -4 & -12 & \underline{0} & 12 & 4 \\ -6 & -8 & 0 & 8 & 1 \\ -4 & -1 & -2 & 0 & 2 \end{bmatrix}$$

$$[G_{225^\circ}] = \begin{bmatrix} 0 & 2 & 1 & 4 & 6 \\ -2 & 0 & 8 & 12 & 4 \\ -1 & -8 & \underline{0} & 8 & 1 \\ -4 & -12 & -8 & 0 & 2 \\ -6 & -4 & -1 & -2 & 0 \end{bmatrix}$$

$$[G_{243^\circ}] = \begin{bmatrix} 2 & 1 & 4 & 6 & 4 \\ 0 & 0 & 8 & 12 & 1 \\ -2 & -8 & \underline{0} & 8 & 2 \\ -1 & -12 & -8 & 0 & 0 \\ -4 & -6 & -4 & -1 & -2 \end{bmatrix}$$

$$[G_{270^\circ}] = \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 2 & 8 & 12 & 8 & 2 \\ 0 & 0 & \underline{0} & 0 & 0 \\ -2 & -8 & -12 & -8 & -2 \\ -1 & -4 & -6 & -4 & -1 \end{bmatrix}$$

$$[G_{297^\circ}] = \begin{bmatrix} 4 & 6 & 4 & 1 & 2 \\ 1 & 8 & 12 & 8 & 0 \\ 2 & 0 & \underline{0} & 0 & -2 \\ 0 & -8 & -12 & -8 & -1 \\ -2 & -1 & -4 & -6 & -4 \end{bmatrix}$$

$$[G_{315^\circ}] = \begin{bmatrix} 6 & 4 & 1 & 2 & 0 \\ 4 & 12 & 8 & 0 & -2 \\ 1 & 8 & \underline{0} & -8 & -1 \\ 2 & 0 & -8 & -12 & -4 \\ 0 & -2 & -1 & -4 & -6 \end{bmatrix}$$

$$[G_{333^\circ}] = \begin{bmatrix} 4 & 1 & 2 & 0 & -2 \\ 0 & 12 & 8 & 0 & -1 \\ 4 & 8 & \underline{0} & -8 & -4 \\ 1 & 0 & -8 & -12 & -6 \\ 2 & 0 & -2 & -1 & -4 \end{bmatrix}$$

$$G_{\psi+180^\circ} = -G_\psi$$

The Sobel's gradient operator in x-direction has a symmetry, namely the following holds for the coefficients of the matrix $[G_{0^\circ}]$:

$$c_{n,m} = -c_{5-n+1,5-m+1}, \qquad n, m = 1,2. \tag{7}$$

It means that the rotation by 180° changes only the sign of the matrix. Therefore,

$$G_{\psi+180^\circ} = -G_{\psi}, \tag{8}$$

for any angle $\psi \in \Psi$. For example, $[G_{180^\circ}] = -[G_{0^\circ}]$ and $[G_{207^\circ}] = -[G_{27^\circ}]$. The number of different oriented gradient images $|G_{\psi_k}(f)|$ for the Sobel's operators is 8.

We thus consider only the first 8 angles ψ_k in the set Ψ, i.e., the set

$$\Psi_1 = \{0^\circ, 27^\circ, 45^\circ, 63^\circ, 90^\circ, 117^\circ, 135^\circ, 153^\circ\}. \tag{9}$$

The full set of the oriented gradient operators is

$$G_\Psi = \{G_{0^\circ}, G_{27^\circ}, G_{45^\circ}, G_{63^\circ}, G_{90^\circ}, G_{117^\circ}, G_{135^\circ}, G_{153^\circ}\}. \tag{10}$$

The angles for the Sobel's gradient operator : $\Psi_1 = \{0^\circ, 27^\circ, 45^\circ, 63^\circ, 90^\circ, 117^\circ, 135^\circ, 153^\circ\}$

$|G_{0^\circ}(f)|$ $|G_{27^\circ}(f)|$ $|G_{45^\circ}(f)|$ $|G_{63^\circ}(f)|$

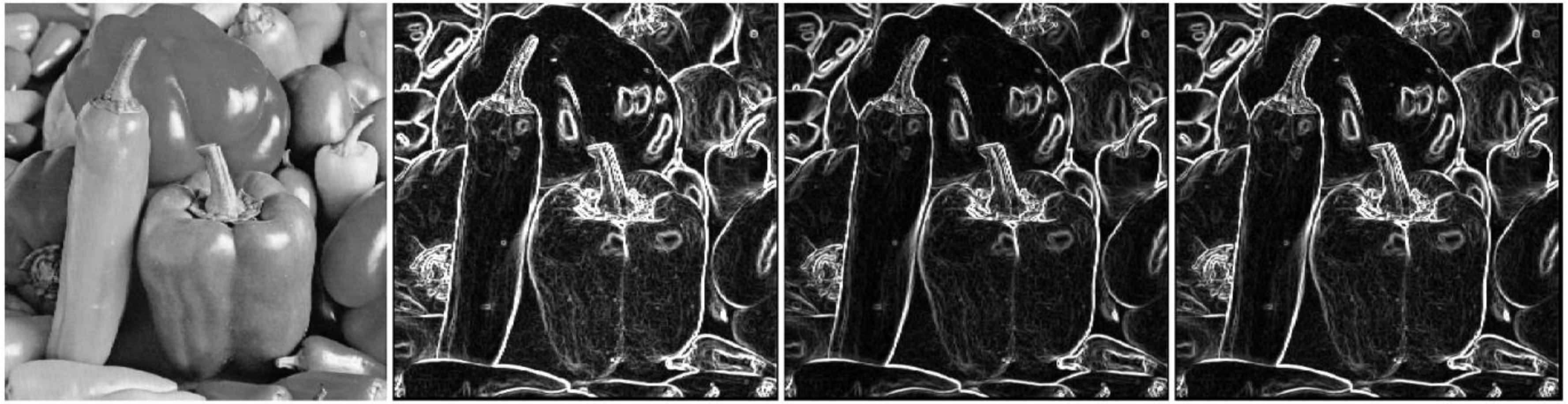
$|G_{90^\circ}(f)|$ $|G_{117^\circ}(f)|$ $|G_{135^\circ}(f)|$ $|G_{153^\circ}(f)|$

Fig. 10 Oriented gradient images $|G_{\psi_k}(f)|$, for 8 angles $\psi_k \in \Psi$, $k = 1{:}8$.

The Sobel's gradients: the maximum, square-root, and magnitude gradient images

$$G_{\mathrm{m}}(f) = \max\left\{|G_{\varphi_{\mathrm{k}}}(f)|; k = 1{:}\,8\right\}, \qquad (11)$$

$$G^2(f) = \sqrt{\frac{1}{8}\sum_{k=1}^{8}\left[G_{\varphi_{\mathrm{k}}}(f)\right]^2}, \qquad G(f) = \frac{1}{8}\sum_{k=1}^{8}|G_{\varphi_{\mathrm{k}}}(f)|. \qquad (12)$$



(a) image (b) maximum (c) sum (d) square-root

Fig. 11 (a) The "pepper" image, and (b) the Sobel' maximum, (b) magnitude, and (c) square-root gradient images composed from gradients in 8 directions.

THE 8-ANGLE AGAIAN's ORIENTED GRADIENTS

The matrix of the 9-level Agaian-Frei-Chen's differencing operator in the horizontal direction is [without the scale factor 1/(8+6$\sqrt{2}$)]

$$[G_x] = \begin{bmatrix} 1 & a & 0 & -a & -1 \\ a & 2 & 0 & -2 & -a \\ 2 & 2a & 0 & -2a & -2 \\ a & 2 & 0 & -2 & -a \\ 1 & a & 0 & -a & -1 \end{bmatrix} = \begin{bmatrix} 1 & \sqrt{2} & 0 & -\sqrt{2} & -1 \\ \sqrt{2} & 2 & 0 & -2 & -\sqrt{2} \\ 2 & \sqrt{8} & 0 & -\sqrt{8} & -2 \\ \sqrt{2} & 2 & 0 & -2 & -\sqrt{2} \\ 1 & \sqrt{2} & 0 & -\sqrt{2} & -1 \end{bmatrix}, \qquad (13)$$

where $a = \sqrt{2}$.

This matrix, as the matrices of the Sobel's and Prewitt gradient operators in x-direction, has similar columns from the left and right side of the middle column. Therefore, the oriented gradients will be calculated only for eight angles $0^{\circ}, 27°, 45^{\circ}, 63°, 90^{\circ}, 117°, 135^{\circ}$, and $153°$. The rotation of this matrix by the set of 8 angles results in the following matrices of the gradients:

The matrices of the eight oriented gradients for angles $\mathbf{0^\circ, 27^\circ, 45^\circ, 63^\circ, 90^\circ, 117^\circ, 135^\circ}$, and $\mathbf{153^\circ}$.

$[G_{0^\circ}]$

$$= \begin{bmatrix} 1 & a & 0 & -a & -1 \\ a & 2 & 0 & -2 & -a \\ 2 & 2a & 0 & -2a & -2 \\ a & 2 & 0 & -2 & -a \\ 1 & a & 0 & -a & -1 \end{bmatrix}$$

$[G_{27^\circ}]$

$$= \begin{bmatrix} a & 0 & -a & -1 & -a \\ 1 & 2 & 0 & -2 & -2 \\ a & 2a & 0 & -2a & -a \\ 2 & 2 & 0 & -2 & -1 \\ a & 1 & a & 0 & -a \end{bmatrix}$$

$[G_{45^\circ}]$

$$= \begin{bmatrix} 0 & -a & -1 & -a & -2 \\ a & 0 & -2 & -2a & -a \\ 1 & 2 & 0 & -2 & -1 \\ a & 2a & 2 & 0 & -a \\ 2 & a & 1 & a & 0 \end{bmatrix}$$

$[G_{63^\circ}]$

$$= \begin{bmatrix} -a & -1 & -a & -2 & -a \\ 0 & 0 & -2 & -2a & -1 \\ a & 2 & 0 & -2 & -a \\ 1 & 2a & 2 & 0 & 0 \\ a & 2 & a & 1 & a \end{bmatrix}$$

$[G_{90^\circ}]$

$$= \begin{bmatrix} -1 & -a & -2 & -a & - \\ -a & -2 & -2a & -2 & - \\ 0 & 0 & 0 & 0 & \text{[illegible]} \\ a & 2 & 2a & 2 & \text{[illegible]} \\ 1 & a & 2 & a & \text{[illegible]} \end{bmatrix}$$

$[G_{117^\circ}]$

$$= \begin{bmatrix} -a & -2 & -a & -1 & -a \\ -1 & -2 & -2a & -2 & 0 \\ -a & 0 & 0 & 0 & a \\ 0 & 2 & 2a & 2 & 1 \\ a & 1 & a & 2 & a \end{bmatrix}$$

$[G_{135^\circ}]$

$$= \begin{bmatrix} -2 & -a & -1 & -a & 0 \\ -a & -2a & -2 & 0 & a \\ -1 & -2 & 0 & 2 & 1 \\ -a & 0 & 2 & 2a & a \\ 0 & a & 1 & a & 2 \end{bmatrix}$$

$[G_{153^\circ}]$

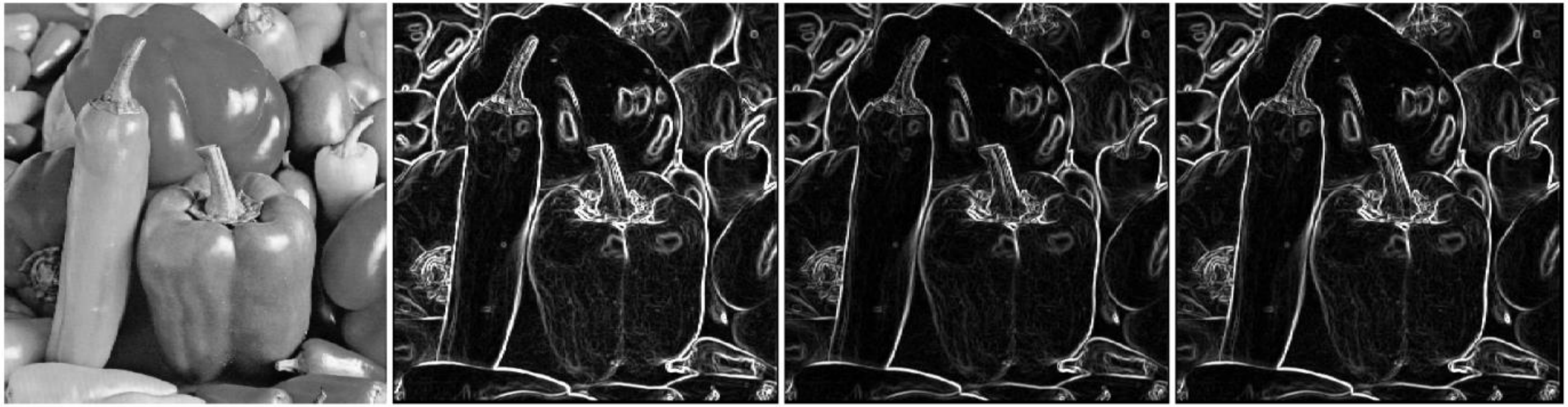
$$= \begin{bmatrix} -a & -1 & -a & 0 & a \\ -2 & -2a & -2 & 0 & 1 \\ -a & -2 & 0 & 2 & a \\ -1 & 0 & 2 & 2a & 2 \\ -a & 0 & a & 1 & a \end{bmatrix}$$

$|G_{0^\circ}(f)|$ $|G_{27^\circ}(f)|$ $|G_{45^\circ}(f)|$ $|G_{63^\circ}(f)|$

$|G_{90^\circ}(f)|$ $|G_{117^\circ}(f)|$ $|G_{135^\circ}(f)|$ $|G_{153^\circ}(f)|$

Figure 12. Oriented gradient images $|G_{\psi_k}(f)|$, for eight angles $\psi_k \in \Psi$, $k = 1{:}8$.

Figure 12 shows the 9-level Agaian-Frei-Chen’s gradient images that are composed from these eight oriented gradients for the “pepper” image, by using Eq. 8.



(a) image (b) maximum (c) sum (d) square-root

Figure 13. (a) The image, and (b) the Agaian-Frei-Chen’s maximum, (c) magnitude, and (d) square-root gradient images composed from gradients in 8 directions.

(a) maximum (b) sum (c) square-root

Figure 14. The Agaian-Frei-Chen's gradient images: (a) maximum, (b) magnitude, and (d) square-root that are composed from gradients in 8 directions.

THE 16-ANGLE ART-SOBEL'S ORIENTED GRADIENTS

Many matrices of the gradient operators in the x-direction are symmetric or symmetric up to the sign with respect to the middle row. Therefore, among 16 rotated gradient matrices, only parts of them are different.

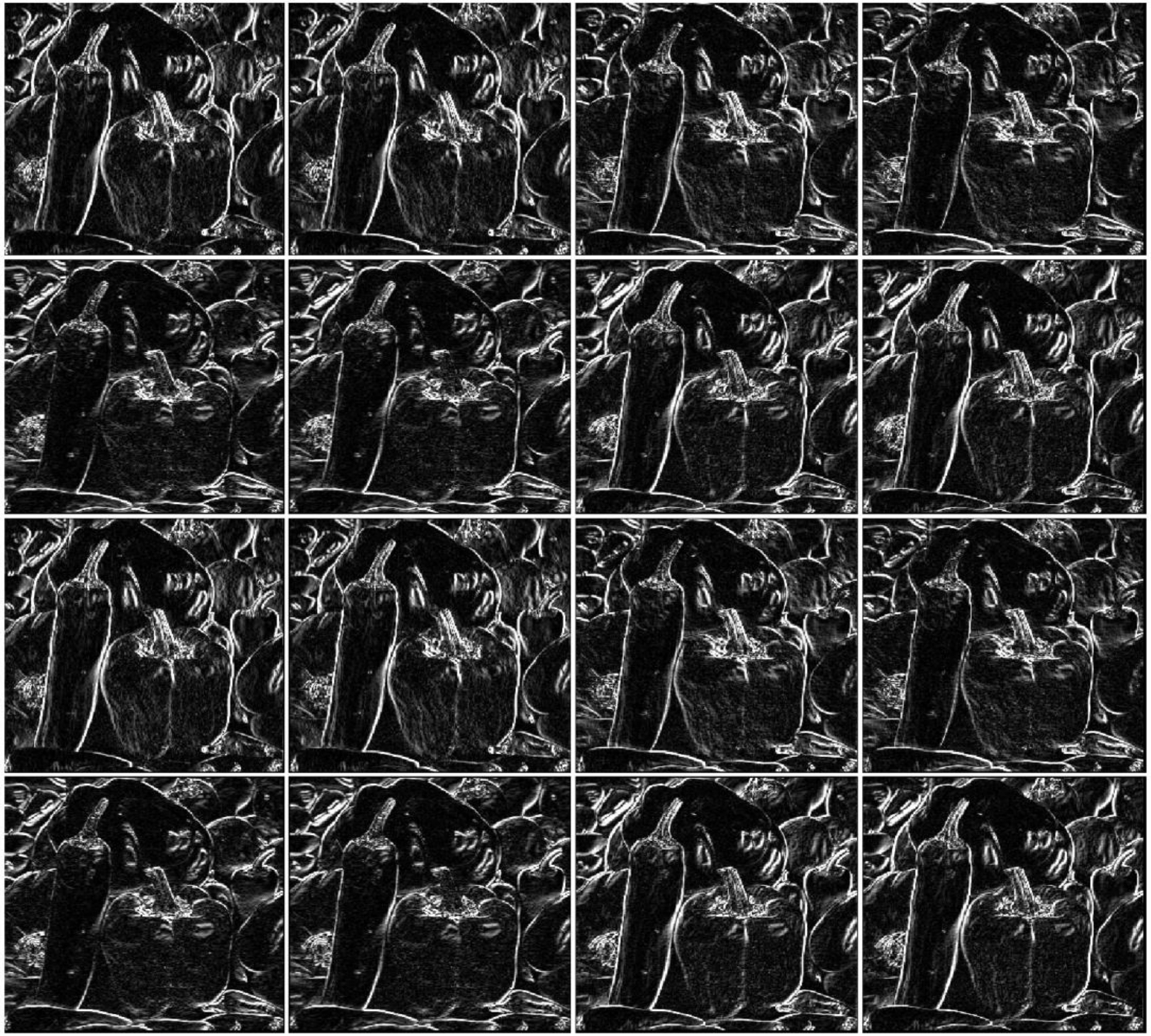
The matrices of the gradients, which are equal up to the sign, result in the same gradient images that are used only in the absolute scale.

The Sobel's and Prewitt gradient operators, which were considered above, are examples of operators with such symmetry. It is clear that these matrices can be modified to remove the property of symmetry. As an example, first we consider the Sobel's gradient.

We can change the Sobel's differencing operator in the first two columns as shown below

$$\begin{bmatrix} 1 & 2 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & \underline{0} & -12 & -6 \\ 4 & 8 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} {\color{red}2} & {\color{red}1} & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & \underline{0} & -12 & -6 \\ {\color{red}8} & {\color{red}4} & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix}. \qquad (14)$$

Thus, we move from the tradition and simple approach in constructing the gradients, when assumed that the change in intensity from left and right, and in other opposite directions are differentiated in the same way. The non-symmetry in the gradient operators may lead to interesting results in edge and counter detection.



$\|G_{0^\circ}(f)\|$	$\|G_{27^\circ}(f)\|$	$\|G_{45^\circ}(f)\|$	$\|G_{63^\circ}(f)\|$
$\|G_{90^\circ}(f)\|$	$\|G_{117^\circ}(f)\|$	$\|G_{135^\circ}(f)\|$	$\|G_{153^\circ}(f)\|$
$\|G_{180^\circ}(f)\|$	$\|G_{207^\circ}(f)\|$	$\|G_{225^\circ}(f)\|$	$\|G_{243^\circ}(f)\|$
$\|G_{270^\circ}(f)\|$	$\|G_{297^\circ}(f)\|$	$\|G_{315^\circ}(f)\|$	$\|G_{333^\circ}(f)\|$

Figure 15. Oriented gradient images $|G_{\psi_k}(f)|$, for 16 angles $\psi_k \in \Psi$, $k = 1{:}\,16$.

Below are all 16 matrices of the oriented Art-Sobel's gradients (without the scale factor 1/48)

$$[G_{0^\circ}] = \begin{bmatrix} 2 & 1 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & \underline{0} & -12 & -6 \\ 8 & 4 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix}$$

$$[G_{27^\circ}] = \begin{bmatrix} 1 & 0 & -2 & -1 & -4 \\ 2 & 8 & 0 & -8 & -6 \\ 4 & 12 & \underline{0} & -12 & -4 \\ 6 & 4 & 0 & -8 & -1 \\ 8 & 1 & 2 & 0 & -2 \end{bmatrix}$$

$$[G_{45^\circ}] = \begin{bmatrix} 0 & -2 & -1 & -4 & -6 \\ 1 & 0 & -8 & -12 & -4 \\ 2 & 8 & \underline{0} & -8 & -1 \\ 4 & 12 & 4 & 0 & -2 \\ 6 & 8 & 1 & 2 & 0 \end{bmatrix}$$

$$[G_{63^\circ}] = \begin{bmatrix} -2 & -1 & -4 & -6 & -4 \\ 0 & 0 & -8 & -12 & -1 \\ 1 & 8 & \underline{0} & -8 & -2 \\ 2 & 12 & 4 & 0 & 0 \\ 4 & 6 & 8 & 1 & 2 \end{bmatrix}$$

$$[G_{90^\circ}] = \begin{bmatrix} -1 & -4 & -6 & -4 & -1 \\ -2 & -8 & -12 & -8 & -2 \\ 0 & 0 & \underline{0} & 0 & 0 \\ 1 & 8 & 12 & 4 & 2 \\ 2 & 4 & 6 & 8 & 1 \end{bmatrix}$$

$$[G_{117^\circ}] = \begin{bmatrix} -4 & -6 & -4 & -1 & -2 \\ -1 & -8 & -12 & -8 & 0 \\ -2 & 0 & \underline{0} & 0 & 2 \\ 0 & 8 & 12 & 4 & 1 \\ 1 & 2 & 4 & 6 & 8 \end{bmatrix}$$

$$[G_{135^\circ}] = \begin{bmatrix} -6 & -4 & -1 & -2 & 0 \\ -4 & -12 & -8 & 0 & 2 \\ -1 & -8 & \underline{0} & 4 & 1 \\ -2 & 0 & 8 & 12 & 8 \\ 0 & 1 & 2 & 4 & 6 \end{bmatrix}$$

$$[G_{153^\circ}] = \begin{bmatrix} -4 & -1 & -2 & 0 & 2 \\ -6 & -12 & -8 & 0 & 1 \\ -4 & -8 & \underline{0} & 4 & 8 \\ -1 & 0 & 8 & 12 & 6 \\ -2 & 0 & 1 & 2 & 4 \end{bmatrix}$$

$$[G_{180^\circ}] = \begin{bmatrix} -1 & -2 & 0 & 2 & 1 \\ -4 & -8 & 0 & 4 & 8 \\ -6 & -12 & \underline{0} & 12 & 6 \\ -4 & -8 & 0 & 8 & 4 \\ -1 & -2 & 0 & 1 & 2 \end{bmatrix}$$

$$[G_{207^\circ}] = \begin{bmatrix} -2 & 0 & 2 & 1 & 8 \\ -1 & -8 & 0 & 4 & 6 \\ -4 & -12 & \underline{0} & 12 & 4 \\ -6 & -8 & 0 & 8 & 2 \\ -4 & -1 & -2 & 0 & 1 \end{bmatrix}$$

$$[G_{225^\circ}] = \begin{bmatrix} 0 & 2 & 1 & 8 & 6 \\ -2 & 0 & 4 & 12 & 4 \\ -1 & -8 & \underline{0} & 8 & 2 \\ -4 & -12 & -8 & 0 & 1 \\ -6 & -4 & -1 & -2 & 0 \end{bmatrix}$$

$$[G_{243^\circ}] = \begin{bmatrix} 2 & 1 & 8 & 6 & 4 \\ 0 & 0 & 4 & 12 & 2 \\ -2 & -8 & \underline{0} & 8 & 1 \\ -1 & -12 & -8 & 0 & 0 \\ -4 & -6 & -4 & -1 & -2 \end{bmatrix}$$

$$[G_{270^\circ}] = \begin{bmatrix} 1 & 8 & 6 & 4 & 2 \\ 2 & 4 & 12 & 8 & 1 \\ 0 & 0 & \underline{0} & 0 & 0 \\ -2 & -8 & -12 & -8 & -2 \\ -1 & -4 & -6 & -4 & -1 \end{bmatrix}$$

$$[G_{297^\circ}] = \begin{bmatrix} 8 & 6 & 4 & 2 & 1 \\ 1 & 4 & 12 & 8 & 0 \\ 2 & 0 & \underline{0} & 0 & -2 \\ 0 & -8 & -12 & -8 & -1 \\ -2 & -1 & -4 & -6 & -4 \end{bmatrix}$$

$$[G_{315^\circ}] = \begin{bmatrix} 6 & 4 & 2 & 1 & 0 \\ 8 & 12 & 8 & 0 & -2 \\ 1 & 4 & \underline{0} & -8 & -1 \\ 2 & 0 & -8 & -12 & -4 \\ 0 & -2 & -1 & -4 & -6 \end{bmatrix}$$

$$[G_{333^\circ}] = \begin{bmatrix} 4 & 2 & 1 & 0 & -2 \\ 6 & 12 & 8 & 0 & -1 \\ 8 & 4 & \underline{0} & -8 & -4 \\ 1 & 0 & -8 & -12 & -6 \\ 2 & 0 & -2 & -1 & -4 \end{bmatrix}$$

Comparison: 4, 8, and 16 gradient operators

4 directions 8 directions 16 directions

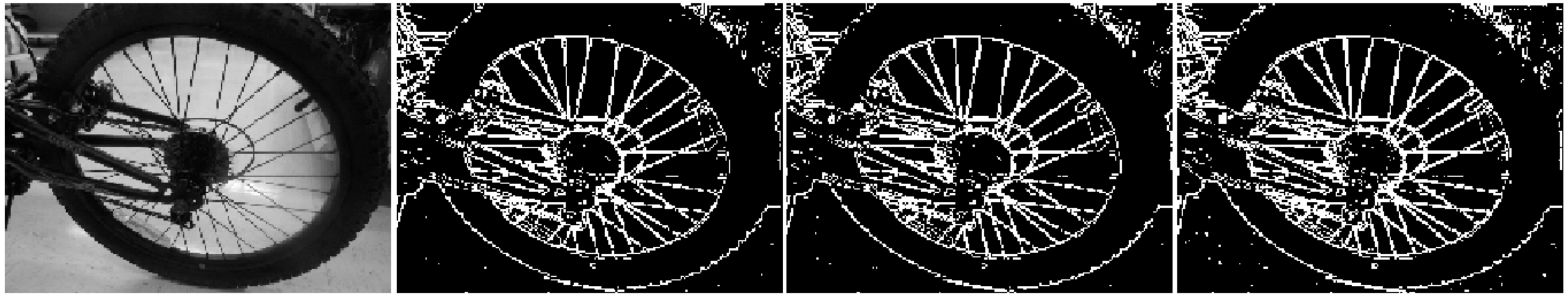
Magnitude gradients

4 directions 8 directions 16 directions

Maximum gradients

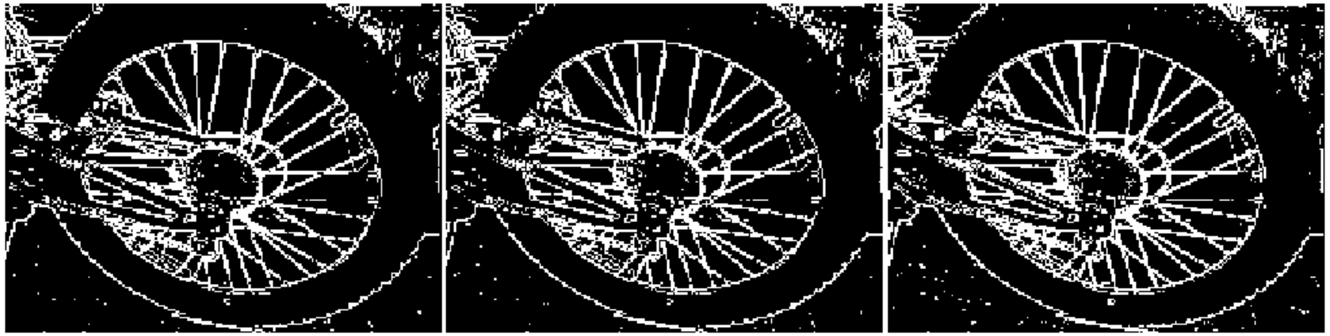
Figure 16. The “pepper” image and Art-Sobel maximum and magnitude gradients by 4, 8, and 16 directions, after thresholding by T=12.

Comparison: 4, 8, and 16 gradient operators



Magnitude gradients

4 directions 8 directions 16 directions



Maximum gradients

4 directions 8 directions 16 directions

Figure 17. The “bike-wheel” image and Art-Sobel maximum and magnitude gradients by 4, 8, and 16 directions after thresholding.

SUMMARY

We present the model of rotations to obtain gradient operators that are defined along maximum of 16 directions and compose the campus gradients. The different examples of gradients were considered, including the Sobel, Prewitt, Agaian, and Nevatia-Babu operators. Each gradient is defined by the mask, which is initially taken along X-direction. The number of directions in campus gradients for each operator is determined separately. For instance, for the Nevatia-Babu gradient, the campus gradient is composed by six different oriented gradients.

With mask larger than 3×3, for instance 5×5, the campus gradient is composed with more than eight oriented gradients. For instance, the for the Art-Sobel gradient operator, the compass gradient is composed from 16 different oriented gradient operators.

REFERENCES

1. A.M. Grigoryan, S.S. Agaian, "Two general models for gradient operators in imaging," Proceedings of IS&T Intern. Symposium, Electronic Imaging: Algorithms and Systems, 28 Jan.-2 Feb., Burlingame, CA, (2018).
2. A.M. Grigoryan, S.S. Agaian, "Asymmetric and symmetric gradient operators with application in face recognition in Renaissance portrait art," *Proc. of SPIE, Defense + Commercial Sensing, Mobile Multimedia/Image Processing, Security, and Applications 2019, vol. 10993, p. 12, Maryland, (16 April 2019).*
3. A.M. Grigoryan, S.S. Agaian, Quaternion and Octonion Color Image Processing with MATLAB, SPIE PRESS, vol. PM279 (2018). https://doi.org/10.1117/3.2278810
4. A.M. Grigoryan, S.S. Agaian, "Color facial image representation with new quaternion gradients," Image Processing: Algorithms and Systems, IS&T Electronic Imaging Symposium, p. 6, (2018).