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A Novel Approach to State-to-State Transformation in Quantum Computing

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INTRODUCTION

A novel technique using the Quantum Signal Induced Heap Transform (QsiHT) is presented for preparing arbitrary real amplitude quantum states and transforming one quantum state into another. We show that a r-qubit superposition $|\boldsymbol{x}\rangle$ can be obtained from another r-qubit superposition $|\boldsymbol{y}\rangle$, by using only $(2^r\!-\!1)$ controlled rotation gates. The traditional two-stage approach $U_y^{-1}U_x\colon |\boldsymbol{x}\rangle \to |0\rangle^{\oplus r} \to |\boldsymbol{y}\rangle$ requires twice as many rotations.

The QsiHT only uses controlled elementary rotation gates around the y −axis with a recursive form for any arbitrary r-qubit superposition where $r \geq 2$.

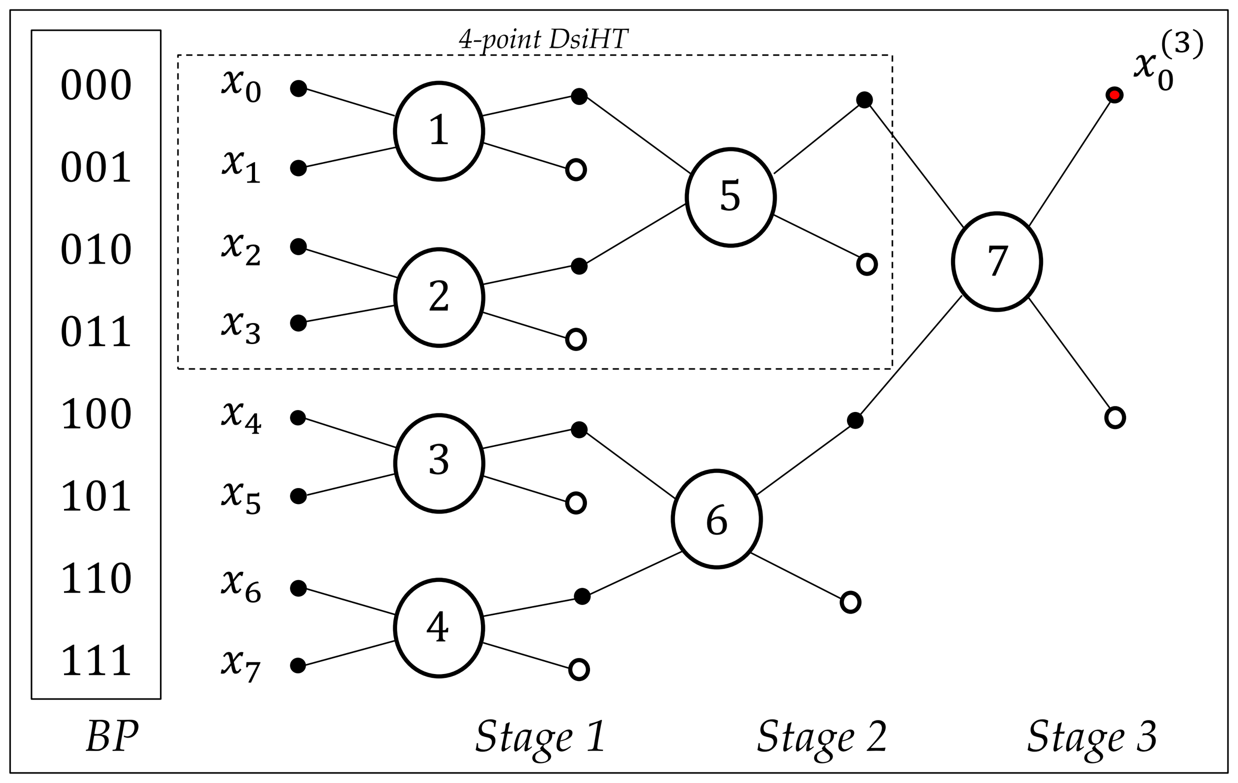

Fig. 1 The 8-point DsiHT used to construct the 3-qubit QsiHT fast path with recursiveness.

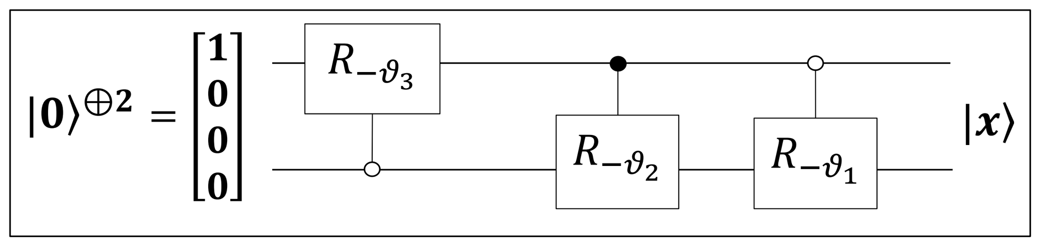

Fig. 2 The circuit for the initiation of any arbitrary 2-qubit state $|\boldsymbol{x}\rangle$ from $|0\rangle^{\oplus 2}$ using the QsiHT fast pass.

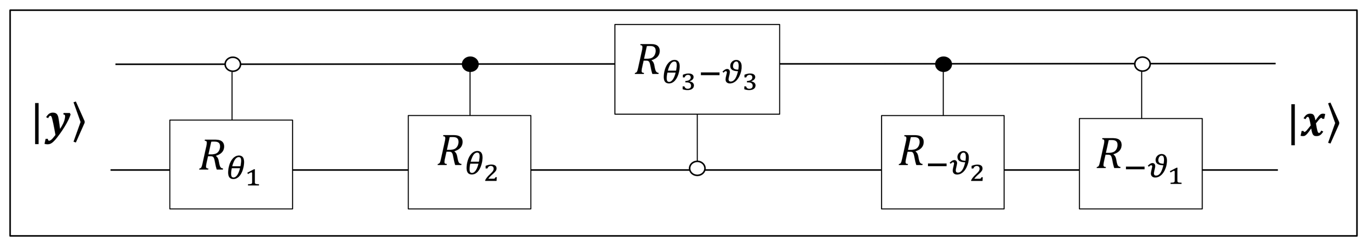

Fig. 3 The circuit for the initiation of the 2-qubit state $|\boldsymbol{x}\rangle$ from $|\boldsymbol{y}\rangle$ through the QsiHT fast pass

The main contributions of this work are as follows:

- Proposes the Quantum Signal Induced Heap Transform (QsiHT) that enables r-qubit state-to-state transformations using only $(2^r - 1)$ controlled Ry rotation gates that halves the number of gates required compared to prior methods.
- Demonstrates recursive and simple circuits for 2–4 qubits without the need for permutations.
- Outlines fast paths in the QsiHT that reduce complexity, which offers scalable and efficient preparation on near-term quantum devices.

METHODS

State preparation and transfer circuits are designed through the Digital signal-induced Heap Transform (DsiHT) using a chosen path as shown Fig. 1. Using the quantum analogue of the DsiHT, the QsiHT is used to realize the corresponding quantum circuit with the procedure for calculating the generator's angles as shown in Fig. 2. The generator specifies the quantum states to be prepared along the selected path. These states are parameterized by the angles, which uniquely define the transformations required for the corresponding path.

The DsiHT with the Strong-Wheel Carriage Path Example:

The case of real signals is considered. In this example, on the first step, the angle of the rotation is calculated from the conditions

$$T_1\begin{bmatrix}x_6\\x_7\end{bmatrix}=\begin{bmatrix}x_0^{(1)}\\0\end{bmatrix}\quad\text{and}\quad |x_0^{(1)}|=\sqrt{x_6^2+x_7^2}.$$

This transform is the Givens rotation calculated by

$$T_1=T_{\varphi_1}=\begin{bmatrix}\cos\vartheta_1 & -\sin\vartheta_1\\ \sin\vartheta_1 & \cos\vartheta_1\end{bmatrix},$$

with the angle $\vartheta_1=-\arctan(x_7/x_6)$. If $x_6=0$, the angle $\vartheta_1=-\pi/2$, or $\pi/2$. Thus, the angle of rotation is defined from the angular equation $x_6\sin\vartheta_1+x_7\cos\vartheta_1=0$. Then, the first output, $x_0^{(1)}$, of the transform is calculated in the first equation. On the next step, the second angle of rotation, $T_2=T_{\vartheta_2}$ is calculated from the conditions

$$T_2\begin{bmatrix}x_5\\x_0^{(1)}\end{bmatrix}=\begin{bmatrix}x_0^{(2)}\\0\end{bmatrix}\ \text{and}\ |x_0^{(2)}|=\sqrt{x_5^2+\left(x_0^{(1)}\right)^2}.$$

Thus, $\vartheta_2=-\arctan\left(x_5/x_0^{(1)}\right)$. Continuing similar calculations, the last angle $\vartheta_7=-\arctan\left(x_0^{(6)}/x_0\right)$ of rotation $T_7=T_{\varphi_7}$ is calculated and then the last value $x_0^{(7)}$,

$$T_7\begin{bmatrix}x_0\\x_0^{(6)}\end{bmatrix}=\begin{bmatrix}x_0^{(7)}\\0\end{bmatrix}.$$

The value $x_0^{(7)}$ contains information of the energy of the generator.

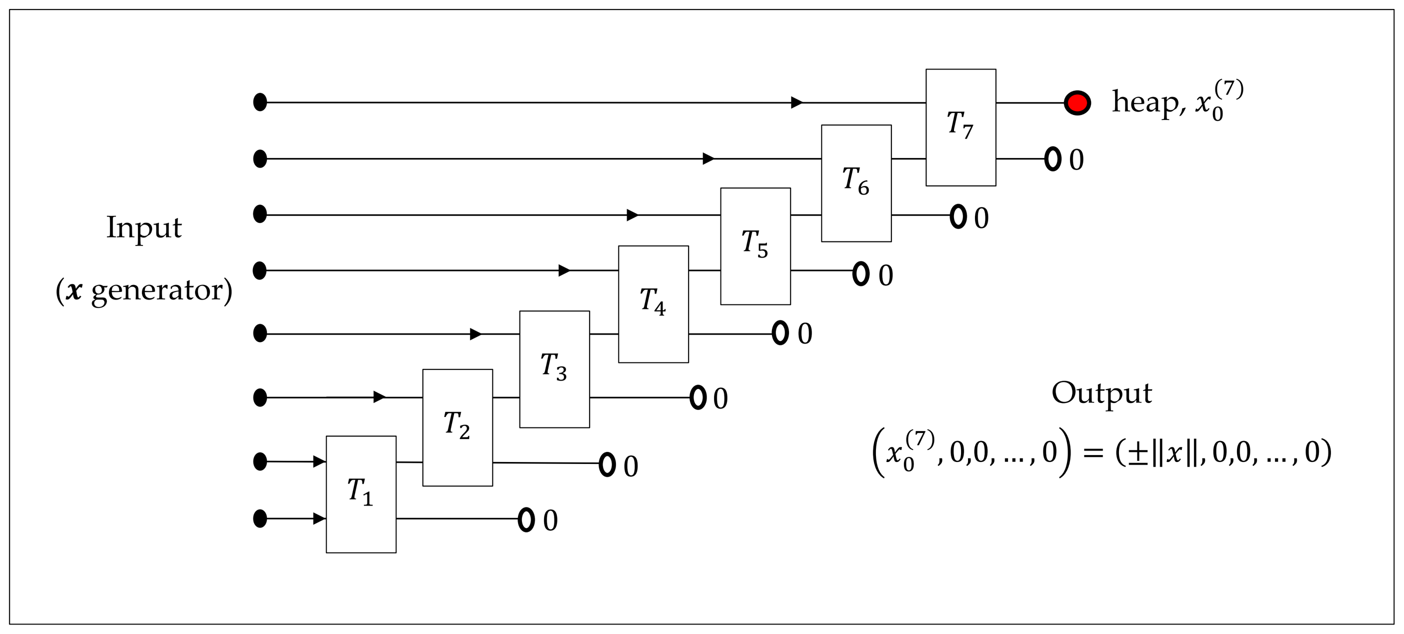

Fig. 5 Visualization of the 8-point strong-path DsiHT of the generator.

This process can be extended for any r-qubit by properly scaling the number of stages for the QsiHT state preparation and transfer algorithm.

RESULTS

The QsiHT using the fast path for state preparation and state transfer allows for simpler, permutation-free circuits that significantly reduce computational complexity and mean-square root error (MSRE) in theoretical and noise-free quantum simulations. The construction of the quantum algorithms presented in this work and their coherency validation are done through IBM's Qiskit Python Framework.

Under theoretical and noise-simulated quantum channel conditions, the QsiHT with fast path allows for rapid convergence onto the desired state.

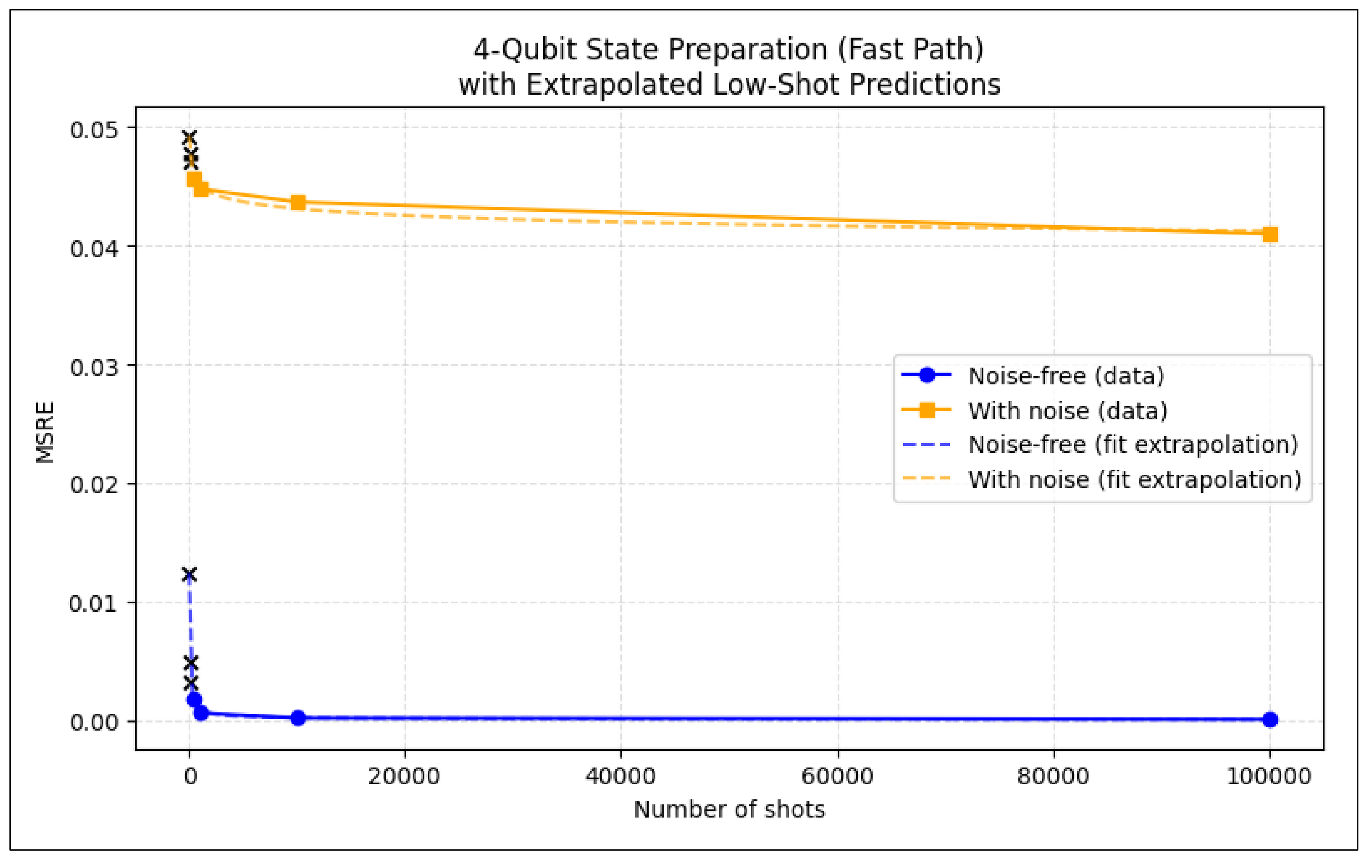

Fig. 6 MRSE amplitude error for preparing $\boldsymbol{x}=(1,-1,2,4,2,-1,2,6,1,1,4,-2,4,-1,1,2)/\sqrt{111}$ using the QsiHT fast pass.

CONCLUSIONS

1. **The QsiHT enables efficient r −qubit state-to-state transformations with only (2^r-1) rotation gates, reducing the computational overhead compared to existing methods.**
2. **The flexibility of path selection in the QsiHT allows for constructing simple quantum circuits and identifying fast paths that improve performance for multi-qubit superpositions.**
3. **Simulation results confirm that the proposed circuits accurately realize target amplitudes, with sampled probabilities converging toward theoretical values as the number of shots increases.**

REFERENCES

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