THE UNIVERSITY OF TEXAS AT SAN ANTONIO
EE 5143
LINEAR SYSTEMS AND CONTROL

HOMEWORK # 2
Ahmad F. Taha
September 26, 2017

The objective of this homework is to test your understanding of the content of Module 2. Due date of the homework is: Thursday, September 7th, 2017 @ 23:59pm. You have to upload either a **clear** scanned version of your solutions on Blackboard or a typed PDF via LaTeX. For your convenience, I am also attaching the LaTeX source files.

---

1. Go through the following links:
   (a) State space on MATLAB:
   http://www.mathworks.com/videos/state-space-models-part-1-creation-and-analysis-100815.html
   (b) Simulating state space using ODE solver: https://www.mathworks.com/matlabcentral/answers/146782-solve-state-space-equation-by-ode45. Go through the first response only.

2. In this link http://academic.csuohio.edu/richter_h/courses/mce371/mce371_5.pdf, you'll find a quick introduction to state space and its implementation on MATLAB, similar to the one above.
   (a) Go through Pages 3–14 of this PDF presentation. Make sure that you understand the details involved.
   (b) You are now given the following dynamical system (identical to the one given in the PDF):
   $$2y^{(4)}(t) + 0.9y^{(3)}(t) + 45.1\ddot{y}(t) + 10\dot{y}(t) + 250y(t) = 250u(t),$$
   where $y(t)$ and $u(t)$ are the output and input to the system. Derive **two different** state space representations, i.e., **obtain two sets** of state-space matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ for this fourth order ODE.
   (c) For each set of the derived matrices, and given what you learned about the ODE solvers on MATLAB for state-space systems, simulate the dynamics of this system assuming that the input $u(t)$ is a unit step function ($u(t) = 1$). Consider that the time horizon is equal to 2 seconds. You'll have to plot the states of the system with respect to time, as well as the output $y(t)$. You can assume zero initial conditions.
   (d) Is there a difference between the output and the states for the two state-space representations? Why/Why Not? Explain your answer.
   (e) Find the transfer function associated to the two distinct state space representations that you derived in 2-(b). Is the transfer function unique? Why/Why Not?

3. Assume that two systems, N_1 and N_2, are cascaded in series. System N_1 is defined by the derivative operator (i.e., the output to N_1 is the derivative of its input), and system N_2 is defined by a function $\gamma(t)$ (i.e., the output of N_2 is the input of N_2 multiplied by $\gamma(t)$). Is the overall, cascaded system linear? Nonlinear? Time-varying? Time-invariant? Prove it.

4. What happens when two systems, out of which one is LTI while the other is LTV, are connected together in series? Would the cascaded overall system still remain linear?

   Now suppose that the order of the cascading is reversed, does that change the overall system output given the same input? Yes/No answers do not suffice. You have to prove your result.

5. Assume that a system N is linear, with an output defined as $y(t) = N(u(t))$. Prove that if the input to the system is zero for all $t \geq 0$, then the output must be also 0 for all $t \geq 0$.

6. A Trump-Obama dynamical system that exists nowhere follows these two differential equations:

$$\ddot{T}(t) + \alpha_1(t)\dot{T}(t) - \alpha_2(t)\dot{C}(t) = \alpha_3(t)u(t) \quad (1)$$
$$\dot{C}(t) = \alpha_4(t)u(t) - C(t) - \alpha_5(t)T(t), \quad (2)$$

where $T(t)$ and $C(t)$ are the two mental **states** of Jalyooka Trump and Palyooka Obama, $u(t)$ is the control input, and $\alpha_i(t)$ functions are all time-varying functions. Derive the state-space representation of this surely non-existent dynamical system. You should be able to obtain an equation similar to this:

$$\dot{\mathbf{x}}(t) = \boldsymbol{A}(t)\mathbf{x}(t) + \boldsymbol{B}(t)u(t),$$

where $\mathbf{x}(t)$ is the state-vector of the system (minimum of size 3) and $\boldsymbol{A}, \boldsymbol{B}$ are state-space matrices that you should derive (in terms of $\alpha(t)$ functions).

Hint: let $x_1(t) = T(t)$ *an* $x_3(t) = C(t)$.

7. The simplified dynamics of the vertical ascent of a Space X rocket can be modeled as:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -g\left(\frac{D}{x_1(t)+D}\right)^2 + \frac{\ln(u)}{m} \end{bmatrix},$$

where D is the distance from earth to the surface of the rocket (assumed to be constant), m is the actual mass of the rocket, g is the gravity constant, and u is the thrust that is assumed to be constant.

Find the equilibrium states (x_1^*, x_2^*) of the above dynamic system.

8. A transfer function of a linear system is given by:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{2s^3 + 4s + 0.5}{0.5s^3 + 8s^2 + 16s + 22}.$$

Derive the state-space controllable canonical form for this system. You should derive the canonical form and state space matrices, rather than listing them.

9. Is the system defined by

$$y(t) = N(u(t)) = |u(t)| + \alpha(t)u(t)$$

linear or nonlinear? Time varying or time-invariant? Prove your answers.

10. In this problem, we will study the equilibrium of Susceptible-Infectious-Susceptible (SIS) in epidemics—similar to what we discussed in class. The SIS model is appropraite to model viral diseases such as influenza, since recovered people do not grant permanent immunity from being infected again. In other words, you are always susceptible to getting a cold—sad truth, but that's a first world problem.

Anyway, the dynamics of a simplified SIS model can be written as

$$\frac{dS}{dt} = -\frac{\beta SI}{N} + \gamma I \quad (3)$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I \quad (4)$$

where $S(t)$ is the number of people that are susceptible at time t and $I(t)$ is the number of infected people at time t, where N is the total number of people.

Assume that the number of people is fixed, that is $S(t) + I(t) = N$.

(a) Given the above assumption, reduce the above dynamical system from 2 states $(S(t), I(t))$ to a dynamic system with only one state $I(t)$. You should obtain something like

$$I(t) = f(\beta, N, \gamma, I).$$

(b) What is the equilibrium of the system? Analyze the stability of the solution of the first oder ODE of $I(t)$. In other words, explain what happens as $t \to$ as any of these parameters β, N, γ change. Be clear and concise.

$$\begin{aligned}
\underset{\alpha_P, L, n_{as}, n_{exo}, r, c}{\text{minimize}} \quad & W_f |F_{ab} - F_{des}| + W_v |v_{ab} - v_{des}| && \text{(5a)} \\
\text{subject to} \quad & F_{ab} = \sum_{i=1}^{n_{exo}} N_{aas} \cdot 2 && \text{(5b)}
\end{aligned}$$