Stability Analysis of Networked Control Systems

Gregory C. Walsh, Hong Ye, and Linda G. Bushnell

***Abstract*—We introduce a novel control network protocol, try-once-discard (TOD), for multiple-input–multiple-output (MIMO) networked control systems (NCSs), and provide, for the first time, an analytic proof of global exponential stability for both the new protocol and the more commonly used (statically scheduled) access methods. Our approach is to first design the controller using established techniques considering the network transparent, and then to analyze the effect of the network on closed-loop system performance. When implemented, an NCS will consist of multiple independent sensors and actuators competing for access to the network, with no universal clock available to synchronize their actions. Because the nodes act asynchronously, we allow access to the network at anytime but we assume each access occurs before a prescribed deadline, known as the maximum allowable transfer interval. Only one node may access the network at a time. This communication constraint imposed by the network is the main focus of the paper. The performance of the new, TOD protocol and the statically scheduled protocols are examined in simulations of an automotive gas turbine and an unstable batch reactor.**

***Index Terms*—Limited communications, networked control systems (NCSs), stability.**

I. Introduction

IN MANY complicated control systems, such as manufacturing plants, vehicles, aircraft, and spacecraft, serial communication networks are employed to exchange information and control signals between spatially distributed system components, like supervisory computers, controllers, and intelligent input–output (I/O) devices (e.g., smart sensors and actuators). Each of the system components connected directly to the network is denoted as a node. When a control loop is closed via the serial communication channel, we label it a networked control system (NCS). The serial communication channel, which multiplexes signals from the sensors to the controller and/or from the controller to the actuators, serves many other uses besides control (see Fig. 1). In contrast to widely used computer networks, an NCS is concerned primarily with the quality of real-time reliable service.

NCSs are being adopted in many application areas for a number of reasons [16] including their low cost, reduced weight, and power requirements, simple installation and maintenance, and higher reliability. However, using a network presents some new analytical challenges because the network imposes a communication constraint: only one sensor can report its measurements at a time. Furthermore, the lack of a universal clock and the presence of noncontrol related traffic makes assumptions about constant sampling intervals unrealistic in many applications. In this paper, an access deadline, or maximum allowable transfer interval, τ_m, is used in its place to ensure absolute stability of an NCS.

Ample research papers in analyzing and scheduling the real-time network traffic have been published [2], [6], [21], [24], [25]. The significance of combining communication constraints and control specifications has not apparently been addressed in these papers. We propose and analyze a new scheduling algorithm to determine the transmission order of multiple sensor nodes in an NCS based on need. The new scheduling algorithm efficiently allocates network resources to multiple smart sensors and maintains good closed-loop control system performance. Some researchers noticed the detrimental effects of network-induced randomly time-varying delay on the stability of feedback control systems [17], [18]. However, all previous research is confined to the one packet transmission problem, i.e., all system outputs are lumped and sent out in one packet, and as a consequence, there is no competition between smart sensors of an NCS [1], [4], [9], [11], [22], [23]. No general explicit stability condition has been obtained in the literature even for one packet transmission case. This paper presents, for the first time, an analytic proof of global stability for an NCS with general multiple-packet transmission in addition to providing a global stability condition for the special one packet transmission problem.

The augmented state space method and jump linear control system method are two significant methods proposed in the literature for analyzing and designing an NCS. The former one reduced the problem to a finite dimensional discrete-time control by augmenting the system model to include past values of plant

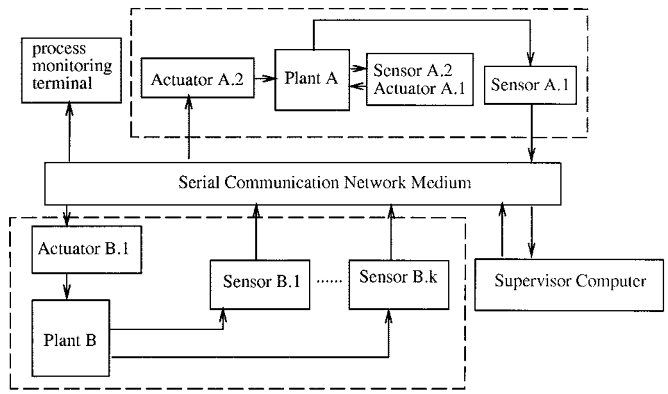

Fig. 1. Schematic diagram of a complicated control system. In this diagram, the network is found between sensors and the controller.

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input and output (i.e., delayed variables) as additional states [4], [5]. A necessary and sufficient condition for system stability was established only for the special case of periodic delays. This technique is very useful for developing control laws to improve the performance of an NCS [12], [13], [19], [20] except that it fails to give a general stability condition for random delay. In [11] and [14], distributed linear feedback control systems with random communication delays were modeled as a jump linear control systems, in which random variation of system delays corresponds to randomly varying structure of the state-space representation. Necessary and sufficient conditions were found for zero-state mean-square exponential stability of the considered class of systems. This method requires that the transition probability matrix is known *a priori*. Furthermore, both methods were limited to the one packet transmission problem.

The occurrence of transmission events on the network is time-varying and often modeled as a random process, e.g., Poisson process, and the resulting times that pass between each access to the network are independent and have an exponential distribution. The stochastic Lyapunov function method [8], [9] holds much promise for determining almost-sure stability and control system performance. This paper's approach, however, is to provide guarantees by employing transmission deadlines. The results presented here are absolute instead of almost sure.

This paper is organized as follows. The dynamic model for the NCS and our new TOD protocol are described in Section II. In particular, we model the effect of different scheduling technique as a finite error bound imposed on the system. In Section III, absolute stability conditions are derived for both a multiple-packet transmission system and a single packet transmission system. The results of numerical simulations are presented in Section IV. Conclusions are stated in Section V.

II. Modeling of a NCS

The NCS model is shown in Fig. 2. It consists of three main parts: the plant $\Sigma_p(A_p, B_p, C_p, 0)$ with state $x_p \in R^{n_p}$ and output $y \in R^{n_r}$; the controller $\Sigma_c(A_c, B_c, C_c, D_c)$ with state $x_c \in R^{n_c}$ and output $u(t) \in R^{n_q}$, and the network, with state $\hat{n}(t) = [\hat{y}(t), \hat{u}(t)]^T$, consisting of the most recently reported versions of $y(t)$ and $u(t)$, Without loss of generality we have assumed $D_p = 0$. Outputs measured locally at an actuator can be incorporated directly into the controller and do not require treatment in our model. If such outputs are needed elsewhere, the actuator node can also be considered a smart sensor. Because of the network, only the reported output $\hat{y}(t)$ is available to the controller and its prediction processes, similarly, only $\hat{u}(t)$ is available to the actuators on the plant. Commonly used local area networks support broadcast, hence $\hat{n}(t)$ is globally known and in such a case the controller itself may be physically distributed.

To focus on the effect of network competition on the stability of an NCS, we make the following assumptions. The control law is designed in advance without considering the presence of the network. The controller dynamics are considered continuous and sampling delay is ignored, because the access interval of the NCS to the network is much larger than the processing period of the controller and smart sensors. Once access to a particular sensor node is granted, data is assumed to be transmitted

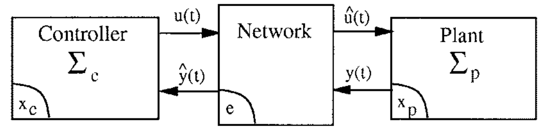

Fig. 2. Configuration of a networked control system.

instantly, since most of the NCS is connected by a local area network with very high data rate and a physical range less than 100 m. The communication medium is error-free based on the lower error rate of modern high-speed communication systems and the higher reliability offered by many error detection and correction technologies. No observation noise exists. All matrices in the paper have compatible dimensions and the standard Euclidean norm will be used unless noted otherwise.

We label the network-induced error $e(t) := \hat{n}(t) - [y(t), u(t)]^T$ and the combined state of the controller and plant $x(t) = [x_p(t), x_c(t)]^T$. The state of the entire NCS is given by $z(t) = [x(t), e(t)]^T$ and between transmission instances the dynamics of the NCS can be summarized as

$$\dot{z}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{1}$$

where

$$A_{11} = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix}$$
$$A_{12} = \begin{bmatrix} B_p D_c & B_p \\ B_c & 0 \end{bmatrix}$$
$$A_{21} = -\begin{bmatrix} C_p & 0 \\ 0 & C_c \end{bmatrix} A_{11}$$
$$A_{22} = -\begin{bmatrix} C_p & 0 \\ 0 & C_c \end{bmatrix} A_{12}.$$

Define the matrix A such that $\dot{z}(t) = Az(t)$. Any prediction or filtering process can be used to improve the estimate of $\hat{n}(t)$. Such predicting and filtering will add extra states and dynamics which we incorporate in matrices A_{21} and A_{22}.

Without a network, $e(t) = 0$, and hence the dynamics reduce to $\dot{x}(t) = A_{11}x(t)$. It is assumed that the controller has been designed ignoring the network, hence A_{11} is Hurwitz. Consequently there exists a unique symmetric positive definite matrix P such that

$$A_{11}^T P + P A_{11} = -I. \tag{2}$$

Define the constants $\sigma_1 = \lambda_{\min}(P)$ and $\sigma_2 = \lambda_{\max}(P)$, ($\lambda =$ eigenvalue). Since we are modeling the network as a perturbation on the system, choosing the right-hand side of (2) equal to $-I$ is desirable for maximizing the tolerable perturbation bound [10, p. 206].

The behavior of the network-induced error $e(t)$ is mainly determined by the architecture of the NCS and the scheduling strategy. In the special case of one-packet transmission, there is only one node transmitting control data on the network, therefore the entire vector $e(t)$ is set to zero at each transmission time. For multiple nodes, transmitting measured outputs $y(t)$ and/or computed inputs $u(t)$, the transmission order of the nodes depends on the scheduling strategy chosen for the NCS. In other words, the scheduling strategy decides which components of $e(t)$ are set to zero at the transmission times. Dynamic and static scheduler are two major scheduling strategies. Both of them will be analyzed for NCS implementation.

A dynamic scheduler determines the network schedule while the system runs. Unlike dynamic scheduling processor time in real-time control, however, the information needed to decide which node should be granted access to the network is not centrally located. Based on the characteristics of the real-time NCS, we propose a novel protocol, try-once-discard (TOD) protocol, which employs dynamic scheduling, allocating network resource based on the need. In TOD, the node with the greatest weighted error from the last reported value will win the competition for the network resource. We call the scheduling technique maximum-error-first (MEF) and the protocol TOD because if a data packet fails to win the competition for network access, it is discarded and new data is used next time. Such a method is vulnerable to noise. In practice, the sensor nodes must employ some sort of filtering to prevent a channel with a large noise signal from dominating the network. This protocol can be realized by using the flexible low-level software implementation and the mature hardware technology of controller area network (CAN), which is specifically designed for bitwise arbitration.

How does the TOD protocol work? Without loss of generality, assume there are p nodes competing, each one may be associated with one or multiple plant inputs and outputs. In the TOD protocol, the priority level of each node's message is proportional to the norm of $e_i(t)$, which is a k-dimensional subvector of $e(t)$ with $k \in [1, n_r+n_q]$ representing the number of plant or controller outputs transmitted by node i. The weights assigned to error signals are assumed already built into the output matrix. At every transmission time, the node with the highest priority (or greatest weighted error) gets transmitted. If two or more messages have equal priority, a prespecified ordering of the nodes will be imposed to resolve the collision.

Today, static scheduling is the most common methodology, in which the order (or pattern) of transmission is decided in advance and fixed during system operation. We label such a scheme *static scheduler*, which is typically implemented by polling or by token ring. Though the schedule is fixed, some nodes may be granted access multiple times before others get any access. If a transmission pattern is of length p, every p consecutive visits form a repeated cycle. In one cycle, all nodes are visited at least once. The pattern length p is called the periodicity of the static scheduler.

In order to characterize the behavior of the scheduling algorithms and their relation with $e(t)$, we introduce a constant β. The existence of β will be proved in later section by the Lipschitz condition of the differential equations (1), i.e., over a short period of time the growth in error $e(t)$ will be bounded by a constant β, which is dependent on the system characteristics and initial conditions. Assume the transmission deadline τ_m exists such that $\|e_i(t+\tau_m) - e_i(t)\| < \beta$. The bound τ_m is defined as the maximum allowable transfer interval used to guarantee the absolute stability of NCS. The size of β does not affect the bound of τ_m. The following two lemmas characterize the scheduling algorithms.

Lemma 1 (Dynamic Scheduler Error Bound): Given a dynamic (TOD) network scheduler starting at time t_0 with p nodes competing, maximum allowable transfer interval τ_m, maximum growth in error in τ_m seconds strictly bounded by $\beta \in (0, \infty)$. Then, for any time $t \geq t_0 + p\tau_m$, $\|e(t)\| < \beta p(p+1)/2$.

Proof: There are at least p transmissions in the interval $[t-p\tau_m, t]$. Let $t_1, \ldots, t_p$ be the last p transmission times with $t_0 \leq t_p < \cdots < t_2 < t_1 \leq t$, $t_1 \geq t - p\tau_m$, and $i_1, \ldots, i_p$ be the nodes that got transmitted at those times, respectively.

Suppose the first $k \in [1, p]$ nodes being transmitted are distinct, and the $(k+1)$th transmitted node was also transmitted before, say at time t_l, $l \in [1, k]$. Then $\|e_{i_j}(t)\| < j\beta$, for $j = 1, \ldots, k$. Since node i_l was transmitted both at t_l and t_{k+1}, we have $\|e_{i_l}(t_l^-)\| < (k+1-l)\beta$ with t_l^- denote the instant right before transmission. By the construction of the dynamic scheduler (TOD), at t_l transmission time, node i_l has the greatest error. As a consequence, $\|e_j(t_l)\| \leq \|e_{i_l}(t_l^-)\| < (k+1-l)\beta$ and $\|e_j(t)\| < (k+1)\beta$, for all $j \neq i_l$, $j \in [1, p]$. So $\|e(t)\| \leq \sum_{j=1}^{p} \|e_i(t)\| < \sum_{j=1}^{k} j\beta + (p-k)(k+1)\beta$.

Since $\max_{1 \leq k \leq p} \{\sum_{j=1}^{k} j\beta + (p-k)(k+1)\beta\} = \beta p(p+1)/2$, when $k = p-1$ or p, we have the worst case error bound for the dynamic scheduler $\|e(t)\| < \beta p(p+1)/2$. □

Lemma 2 (Static Scheduler Error Bound): Given a static network scheduler starting at time t_0, with integer periodicity p, maximum allowable transfer interval τ_m, maximum growth in error in τ_m seconds strictly bounded by $\beta \in (0, \infty)$. Then, for any time $t \geq t_0 + p\tau_m$, $\|e(t)\| < \beta p(p+1)/2$.

Proof: The integer periodicity p allows at most p nodes competing. Assume there are K nodes, $K \in [1, p]$. The scheduler design also implies that each node is visited at least once during every p consecutive transmissions.

At least once cycle (or p transmissions) is completed during the interval $[t-p\tau_m, t]$. Let $t_1, \ldots, t_p$ be the last p transmission times with $t \geq t_1 > \cdots > t_p \geq t_0$, $t_1 \geq t - p\tau_m$, and $i_1, \ldots, i_p$ be the nodes that were transmitted at those times, respectively. Then $\|e_{i_m}(t)\| < m\beta$, for $m = 1, \ldots, p$. Since the set $\{1, \ldots, K\} \subseteq \{i_1, \ldots, i_p\}$, we have $\|e(t)\| \leq \sum_{j=1}^{K} \|e_i(t)\| < \sum_{j=1}^{p} j\beta = \beta p(p+1)/2$. So for any time $t \geq t_0 + p\tau_m$, $\|e(t)\| < \beta p(p+1)/2$. □

The worst case error bound of the dynamic scheduler is the same as that of a special case of the static scheduler, i.e., all p nodes are visited equally. The bound is conservative for both scheduling algorithms, because τ_m represents a deadline. But for the same transmission times distribution, the error bound for dynamic scheduler will be better than that of the static scheduler, because it grants access to the node with the greatest error.

III. Stability of Networked Control Systems

Two stability theorems for general multiple-packet and one-packet transmissions are derived in this section. Both theorems are derived based on Lyapunov's second method and treat the network induced-error term as a vanishing perturbation [10, p. 204].

We first consider the stability of a general multiple-packet transmission NCS with either a static or dynamic (TOD) scheduling algorithm.

Theorem 1 (Stability of a General NCS): Given an NCS whose continuous dynamics are described by (1) with p nodes operating under TOD, or with integer periodicity p under static scheduling, and a maximum allowable transfer interval, τ_m, which is less than the minimum of

$$\frac{\ln(2)}{p\|A\|},\ \frac{1}{4\|A\|\left(\sqrt{\frac{\sigma_2}{\sigma_1}}+1\right)p(p+1)}$$

and

$$\frac{1}{8\sigma_2\sqrt{\frac{\sigma_2}{\sigma_1}}\,\|A\|^2\left(\sqrt{\frac{\sigma_2}{\sigma_1}}+1\right)p(p+1)}.$$

Then, the NCS is globally exponentially stable.

Proof: Consider any initial time t_0 and the associated initial state $z(t_0)=(x(t_0),\, e(t_0))^T$. If $\|z(t_0)\|=0$, then for all $t>t_0$ we have $x(t)=0$ and $e(t)=0$, even if there are no transmissions, since we are at an equilibrium point of the system. For this reason, without loss of generality, we assume $\|z(t_0)\|>0$. At time t_0, nothing is known about the magnitude of the error e. However, at time $t_0+p\tau_m$, at least p transmissions of the network have occurred. For any t in $[t_0,\, t_0+p\tau_m]$, we have from Bellman–Gronwall $\|z(t)\| \le e^{\|A\|p\tau_m}\,\|z(t_0)\| < 2\|z(t_0)\|$ where $\|A\|$ is the induced norm of A. Since we have chosen $\tau_m<\ln(2)/\|A\|p$, we have $e^{\|A\|p\tau_m}<2$. This bound is conservative as it holds even in the case where there are no transmissions of data. At any transmission time $t\in[t_0,\, t_0+p\tau_m]$, we have that $\|e(t^-)\|\ge\|e(t^+)\|$ hence $\|z(t^-)\|\ge\|z(t^+)\|$. The notation t^- and t^+ refers to the limits from the left and right, respectively. Because of this bound, we have that for $t\in[t_0,\, t_0+p\tau_m]$

$$\|\dot e(t)\| \le \|A\|\,\|z(t)\| \le 2\|A\|\,\|z(t_0)\|.$$

Hence the maximum possible growth between transmission in this time interval is strictly bounded by $\beta = 2\tau_m(\sqrt{\sigma_2/\sigma_1}+1)\|A\|\|z(t_0)\|$ as $\sqrt{\sigma_2/\sigma_1}+1\ge 2$, where σ_2/σ_1 is the condition number of P.

Depending on the type of protocol utilized, either Lemma 1 or Lemma 2 may be applied to verify that at time $t_0+p\tau_m$, the error is bounded by $\|e(t_0+p\tau_m)\|<\gamma_1\|z(t_0)\|$ where the constant γ_1 is $\gamma_1=\tau_m(\sqrt{\sigma_2/\sigma_1}+1)\|A\|p(p+1)$. We have selected τ_m so that γ_1 is smaller than both $1/4$ and $1/(8\sigma_2\sqrt{\sigma_2/\sigma_1}\|A\|)$. Furthermore, we have that at time $t_0+p\tau_m$, $\|x(t_0+p\tau_m)\| \le \|z(t_0+p\tau_m)\| < 2\|z(t_0)\|$. Consequently, using $\sigma_1\|x\|^2 \le V(x) \le \sigma_2\|x\|^2$ for $V(x)=x^TPx$ we have that $V(x(t_0+p\tau_m)) < 4\sigma_2\|z(t_0)\|^2 < \gamma_2\|z(t_0)\|^2$ with $\gamma_2=4\sigma_2$. Certainly $\forall\, t\in[t_0,\, t_0+p\tau_m]$, $V(x(t))<\gamma_2\|z(t_0)\|^2$. If at any time t we have $V(x(t))\le\gamma_2\|z(t_0)\|^2$, then $\|x(t)\|\le\gamma_3\|z(t_0)\|$ with the constant γ_3 equal to $\gamma_3=2\sqrt{\sigma_2/\sigma_1}$.

At time $t=t_0+p\tau_m$ we have both $V(x(t))<\gamma_2\|z(t_0)\|^2$ and $\|e(t)\|<\gamma_1\|z(t_0)\|$. We now prove by contradiction that these two conditions imply that for all $t>t_0+p\tau_m$, both conditions hold. If at any time $t\ge t_0+p\tau_m$ any of these two conditions fail, then there exists a time $\hat t > t_0+p\tau_m$ which is the first time either one or both conditions have failed. In interval $[t_0,\, \hat t]$, of minimum length $p\tau_m$ seconds, both conditions are met. The state vector $e(t)$ changes discretely, but always decreases in magnitude when jumping, so at a transition time the bound cannot be jumped over. At time $\hat t$, either $\|e(\hat t)\| = \gamma_1\|z(t_0)\|$ or $V(x(\hat t))=\gamma_2\|z(t_0)\|^2$ or both. Suppose we have the case $V(x(\hat t))=\gamma_2\|z(t_0)\|^2$ and $\|e(\hat t)\|\le\gamma_1\|z(t_0)\|$, then we have $\|x(\hat t)\|\ge\|z(t_0)\|$, and hence

$$\begin{aligned}\dot V &\le -\|x(\hat t)\|^2 + 2\sigma_2\|A_{12}\|\,\|x(\hat t)\|\,\|e(\hat t)\|\\ &\le \|x(\hat t)\|(-\|x(\hat t)\| + 2\sigma_2\|A\|\gamma_1\|z(t_0)\|)\\ &\le \|x(\hat t)\|\left(-\tfrac{3}{4}\right)\|z(t_0)\|\end{aligned} \tag{3}$$

because

$$\gamma_1 < \frac{1}{8\sigma_2\sqrt{\frac{\sigma_2}{\sigma_1}}\,\|A\|} < \frac{1}{8\sigma_2\|A\|}.$$

Since at time $\hat t$, $\|x(\hat t)\|\ge\|z(t_0)\|$, $\dot V$ is strictly negative. Even if $\hat t$ is a transition time, $\|e(t)\|$ can only reduce instantaneously in size hence $\dot V$ will remain strictly negative.

Certainly then at time $\hat t$, $V(x(\hat t)) < \gamma_2\|z(t_0)\|^2$ and therefore $\|x(t)\|\le\gamma_3\|z(t_0)\|$ for all $t\in[t_0,\, \hat t]$ including time $\hat t$. We now consider the remaining possibility that at time $\hat t$, we have $\|e(\hat t)\|=\gamma_1\|z(t_0)\|$. We can conclude $\forall\, t\in[t_0,\, \hat t]$ not a transition time

$$\begin{aligned}\|\dot e(t)\| &\le \|A\|(\|x(t)\|+\|e(t)\|)\\ &\le \|A\|(\gamma_1+\gamma_3)\|z(t_0)\|.\end{aligned}$$

Note that

$$(\gamma_1+\gamma_3) = 2\left(\sqrt{\frac{\sigma_2}{\sigma_1}}+\frac{\gamma_1}{2}\right) < 2\left(1+\sqrt{\frac{\sigma_2}{\sigma_1}}\right)$$

because τ_m has been chosen so that $\gamma_1<1/4$. We now have the conditions of Lemma 1 or Lemma 2 applying to the interval $[t_0,\, \hat t]$, since the maximum growth of $\|e(t)\|$ in τ_m seconds in the interval $[t_0+p\tau_m,\, \hat t]$ is limited by $\tau_m\|\dot e(t)\|<\beta$. The lemmas indicate that $\|e(\hat t)\| < \gamma_1\|z(t_0)\|$, manifesting our contradiction. We conclude then $\forall\, t>t_0+p\tau_m$, $\|e(t)\|<\gamma_1\|z(t_0)\|$.

View the control system as perturbed by the bounded error signal $e(t)$. If we write $\Phi(t,\, t_0+p\tau_m) = e^{A_{11}(t-(t_0+p\tau_m))}$, then the state starting at time $t_0+p\tau_m$ evolves according to the variation of parameters formula:

$$\|x(t)\| \le \|\Phi(t,\, t_0+p\tau_m)\|\,\|x(t_0+p\tau_m)\| + \left\|\int_{t_0+p\tau_m}^{t}\Phi(t,\, w)A_{12}e(w)\,dw\right\|.$$

The zero state term is the solution of the differential equation $\dot{x_z}(t)=A_{11}x_z(t)+A_{12}e(t)$ with zero initial conditions, that is, $x_z(t_0+p\tau_m)=0$. At time $t_0+p\tau_m$ we have $V_z(x_z(t)) = \|x_z(t)\|=0$. We know that for all time, $\|e(t)\|\le\gamma_1\|z(t_0)\|$, consequently, $V_z(x_z(t)) < 4\sigma_2^3\|A\|^2\gamma_1^2\|z(t_0)\|^2$, since if we had equality, then $\|x_z(t)\| \ge 2\sigma_2\|A\|\gamma_1\|z(t_0)\|$ and by (3), $\dot V_z<0$. As $V_z(x_z(t))$ is bounded, then

$$\|x_z(t)\| < 2\sigma_2\sqrt{\frac{\sigma_2}{\sigma_1}}\,\|A\|\gamma_1\|z(t_0)\|.$$

By choice of τ_m, we have $\gamma_1 < (8\sigma_2\sqrt{\sigma_2/\sigma_1}\|A\|)^{-1}$ and therefore the zero state term is strictly less than $(1/4)\|z(t_0)\|$.

Given A_{11} Hurwitz, there exists a time $t_1 > p\tau_m$ such that $\|\Phi(t_1+t_0, t_0+p\tau_m)\| < 1/4$. Finally, $\|e(t)\| < \gamma_1\|z(t_0)\| < 1/4\|z(t_0)\|$ and

$$\|z(t_1+t_0)\| \le \|x(t)\| + \|e(t)\| \le \rho\|z(t_0)\|$$

with $0 < \rho < 3/4$. Then by induction $\|z(kt_1+t_0)\| \le \rho^k\|z(t_0)\|$. The closed-loop NCS is then exponentially stable.□

One-packet transmission is a special case of multiple-packet transmission. In the following, a general sufficient stability condition of an NCS with one-packet transmission is derived in a way different from the above proof. This derivation gives a less conservative bound on τ_m.

In the one-packet transmission case, for $t \in [t_i, t_{i+1})$, $i = 0, 1, 2, \ldots$, $\hat{y}(t) = y(t_i) = C_p x_p(t_i)$, $\hat{u}(t) = u(t_i) = C_c x_c(t_i) + D_c C_p x_p(t_i)$. The system equation can be written as $\dot{x}(t) = A_{11}x(t) + g(x, \hat{x})$ where $g(x, \hat{x}) = A_{12}\left[\begin{smallmatrix} C_p & 0 \\ D_cC_p & C_c\end{smallmatrix}\right](x(t) - x(t_i))$. Under normal operation, as $x \to 0$, $g(x, \hat{x})$ will go to zero because $\hat{x}$ will track x closely. Two lemmas are introduced to prove the theorem. It should be noted that Lemma 3 is a variation of the commonly used Bellman–Gronwall Lemma. The proof of Lemma 4 follows that of Lemma 3.

Lemma 3 (Bellman–Gronwall Lemma [7]): Given $\lambda(t)$ and $k(t)$ nonnegative, piecewise continuous and differentiable functions of time t. If the function $y(t)$ satisfies $y(t) \le \lambda(t) + \int_{t_0}^t k(s)y(s)\,ds$, $\forall t \ge t_0 \ge 0$ then $y(t) \le \lambda(t_0)e^{\int_{t_0}^t k(s)\,ds} + \int_{t_0}^t \dot{\lambda}(s)e^{\int_s^t k(\tau)d\tau}\,ds\ \forall t \ge t_0 \ge 0$.

Lemma 4: Given $\lambda(t)$ and $k(t)$ nonnegative piecewise continuous functions of time t with $\lambda(t)$ differentiable. If the function $y(t)$ satisfies $y(t) \le \lambda(t) + \int_t^{t_f} k(s)y(s)\,ds, \forall t_f \ge t \ge 0$, then $y(t) \le \lambda(t_f)e^{\int_t^{t_f} k(s)ds} - \int_t^{t_f} \dot{\lambda}(s)e^{\int_t^s k(\tau)\,d\tau}\,ds, \forall t_f \ge t \ge 0$.

Proof: Let $z(t) = \lambda(t) + \int_t^{t_f} k(s)y(s)\,ds$, then $z(t)$ is differentiable and $z(t) \ge y(t)$

$$\dot{z}(t) = \dot{\lambda}(t) - k(t)y(t), \quad z(t_f) = \lambda(t_f).$$

Let $v(t) = z(t) - y(t) \ge 0$, then $\dot{z}(t) = -k(t)z(t) + \dot{\lambda}(t) + k(t)v(t)$, whose state transition matrix is $\Phi(t, \tau) = e^{\int_\tau^t -k(s)\,ds} = e^{\int_t^\tau k(s)\,ds}$. Therefore

$$z(t) = \Phi(t, t_f)z(t_f) + \int_{t_f}^t \Phi(t, \tau)[\dot{\lambda}(\tau) + k(\tau)v(\tau)]\,d\tau.$$

Since $\int_{t_f}^t \Phi(t, \tau)k(\tau)v(\tau)\,d\tau \le 0, \forall t_f \ge t$, then $z(t) \le \Phi(t, t_f)z(t_f) + \int_{t_f}^t \Phi(t, \tau)\dot{\lambda}(\tau)\,d\tau$.

Substitute $\Phi(t, \tau)$, then $z(t) \le \lambda(t_f)e^{\int_t^{t_f} k(s)\,ds} - \int_t^{t_f} \dot{\lambda}(s)e^{\int_t^s k(\tau)\,d\tau}\,ds$. Thus $y(t) \le \lambda(t_f)e^{\int_t^{t_f} k(s)\,ds} - \int_t^{t_f} \dot{\lambda}(s)e^{\int_t^s k(\tau)\,d\tau}\,ds, \forall t_f \ge t \ge 0$. □

Theorem 2 (Single-Packet Transmission Stability): Let $x = 0$ be a globally exponentially stable equilibrium point of the nonnetworked system. If the maximum transmission interval τ_m satisfies:

a)
$$\frac{\|D\| \cdot \|A_{11}\| \cdot \|A_{11}+D\|^{-1}\left(e^{\|A_{11}+D\|\tau_m} - 1\right)e^{\|A_{11}+D\|\tau_m}}{1 - \|D\| \cdot \|A_{11}+D\|^{-1}\left(e^{\|A_{11}+D\|\tau_m} - 1\right)} < k;$$

b)
$$1 - \|D\| \cdot \|A_{11}+D\|^{-1}\left(e^{\|A_{11}+D\|\tau_m} - 1\right) > 0$$

where

$$k = \frac{1}{2\sigma_2}, \quad D = A_{12}\begin{bmatrix} C_p & 0 \\ D_cC_p & C_c\end{bmatrix};$$

then the origin is a globally exponentially stable equilibrium point of the NCS.

Proof: When $t \in [t_i, t_{i+1})$, define $e_x(t) = x(t) - x(t_i)$, $e_x(t_i) = 0$. The system equation $\dot{x}(t) = A_{11}x(t) + D[x(t) - x(t_i)]$ can be written as

$$\begin{aligned}\dot{e}_x(t) &= \dot{x}(t) = (A_{11}+D)x(t) - Dx(t_i)\\ &= (A_{11}+D)e_x(t) + A_{11}x(t_i)\end{aligned}$$

then $e_x(t) = \int_{t_i}^t [(A_{11}+D)e_x(w) + A_{11}x(t_i)]\,dw$ and

$$\|e_x(t)\| \le \underbrace{\|A_{11}\| \cdot \|x(t_i)\| \cdot (t-t_i)}_{\lambda(t)} + \int_{t_i}^t \underbrace{\|A_{11}+D\|}_{k(w)}\|e_x(w)\|\,dw$$

where $\|\cdot\|$ stands for vector norm or induced matrix norm.

Using Bellman–Gronwall Lemma 3, we get

$$\|e_x(t)\| \le \lambda(t_i)\exp\left(\int_{t_i}^t \|A_{11}+D\|\,ds\right) + \int_{t_i}^t \|A_{11}\| \cdot \|x(t_i)\|\exp\left(\int_s^t \|A_{11}+D\|\,dw\right)\,ds.$$

Since $\lambda(t_i) = 0$

$$\|e_x(t)\| \le \|A_{11}\| \cdot \|A_{11}+D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\cdot(t-t_i)} - 1\right) \cdot \|x(t_i)\|. \quad (4)$$

Because $Dx(t_i)$ is a constant vector in $t \in [t_i, t_{i+1})$, assume $E = -Dx(t_i)$, then $\dot{x}(t) = (A_{11}+D)x(t) - Dx(t_i) = (A_{11}+D)x(t) + E$. In order to derive the relation between $x(t)$ and $x(t_i)$, we fix the final time t_f and let the initial time t be changeable, i.e., $t_i \le t \le t_f < t_{i+1}$. So $x(t) = x(t_f) + \int_{t_f}^t [(A_{11}+D)x(w) + E]\,dw$

$$\|x(t)\| \le \underbrace{\|x(t_f)\| + \|E\| \cdot (t_f - t)}_{\lambda(t)} + \int_t^{t_f} \underbrace{\|A_{11}+D\|}_{k(w)} \cdot \|x(w)\|\,dw.$$

Using Lemma 4, we get

$$\begin{aligned}\|x(t)\| &\le \|x(t_f)\|\exp\left(\int_t^{t_f} \|A_{11}+D\|\,ds\right) + \int_t^{t_f} \|E\|\exp\left(\int_t^s \|A_{11}+D\|\,dw\right)\,ds\\ &= e^{\|A_{11}+D\|\cdot(t_f-t)} \cdot \|x(t_f)\| + \|E\| \cdot \|A_{11}+D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\cdot(t_f-t)} - 1\right).\end{aligned}$$

Let $t = t_i$, $t_f = t$. Since $\|E\| \le \|D\| \cdot \|x(t_i)\|$, then

$$\|x(t_i)\| \le e^{\|A_{11}+D\|\cdot(t-t_i)} \cdot \|x(t)\| + \|D\| \cdot \|x(t_i)\| \cdot \|A_{11}+D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\cdot(t-t_i)} - 1\right).$$

Let $\tau_m = t - t_i$. Since

$$1 - \|D\| \cdot \|A_{11} + D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\tau_m} - 1\right) > 0$$

then

$$\|x(t_i)\| \le \frac{e^{\|A_{11}+D\|\tau_m}}{1 - \|D\| \cdot \|A_{11} + D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\tau_m} - 1\right)} \cdot \|x(t)\|.$$

Using this and inequalities (4), we derive

$$\|e_x(t)\| \le \frac{\|A_{11}\| \cdot \|A_{11} + D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\tau_m} - 1\right) \cdot e^{\|A_{11}+D\|\tau_m}}{1 - \|D\| \cdot \|A_{11} + D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\tau_m} - 1\right)} \cdot \|x(t)\|.$$

Since $x = 0$ is a globally exponentially stable equilibrium point of the nonnetworked system $\dot{x}(t) = A_{11}x(t)$, there exists a unique symmetric positive definite matrix P, satisfying the Lyapunov equation: $PA_{11} + A_{11}^T P = -I$.

Let $V(x) = x(t)^T P x(t)$ be a Lyapunov function of the nonnetworked system satisfying the following inequalities:

$$\sigma_1\|x\|^2 \le V(x) \le \sigma_2\|x\|^2$$
$$\dot{V}(x) = -x^T x \le -\|x\|^2$$
$$\left\|\frac{\partial V}{\partial x}\right\| = \|2xP^T\| \le 2\sigma_2\|x\|.$$

The derivative of $V(x)$ along the trajectories of the perturbed system satisfies $\dot{V}(x) \le -\|x\|^2 + 2\sigma_2\|x\| \cdot \|g(x, \hat{x})\|$.

Since the equation shown at the bottom of the page is true, then $\dot{V}(x) \le -\{1 - 2\gamma \cdot \sigma_2\}\|x\|^2 \le 0$ for $\gamma < 1/2\sigma_2 = k$. So, when τ_m satisfies the inequalities a) and b), $\dot{V}(x) \le 0$ in any transmission interval, $\dot{V}(x) = 0$ only when $x(t) \equiv 0$.

The origin is a globally exponentially stable equilibrium point of the NCS. □

IV. Simulation Results and Discussion

We explore the application of networking technology to two example systems, an unstable batch reactor and an automotive gas turbine. The network is considered transparent for the purpose of controller design. The two models are taken from the literature and the reported controller is used without modification. The bounds on the maximum allowable transfer interval derived in the theorems are very conservative, as both examples demonstrate. The simulations explore not only better estimates for the bound but also the impact of different packet arrival models on the system performance. Our experience suggests that the commonly used Poisson packet arrival model is unlikely to accurately reflect the traffic on a control network, as most packets are relatively short and frequent, and because TOD control traffic does not use queues. Alternate packet arrival models are compared in the unstable batch reactor simulation. The differences between dynamic and static scheduling are explored in a simulation of an automotive gas turbine, because the analysis also does not differentiate between the approaches. A token ring scheduler alternating access between the two nodes and the dynamic TOD scheduling algorithm are compared, using a Poisson packet arrival model.

A. *Unstable Batch Reactor*

The first example, an unstable batch reactor [3, p. 62], is a coupled two-input–two-output NCS. Based on the linearized process model

$$\dot{x} = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix} x$$

a proportional-plus-integral controller

$$K(s) = \begin{bmatrix} 0 & \dfrac{2s+2}{s} \\ \dfrac{-5s-8}{s} & 0 \end{bmatrix}$$

is designed in advance to stabilize the feedback system and achieve good performance.

Only the system outputs y_1 and y_2 need to be transmitted to the controller via the network, each with its associated node. In the simulations, the network model is placed between the output of the plant and the input of the controller. A Poisson process with mean $1/\tau$ is used to model the packet arrival events. In unit time, the probability of k transmission events occurring is $P(k) = \mu^k e^{-\mu}/k!$, where μ stands for the expectation of number of events occurring in unit time. Its inverse, $\tau = 1/\mu$, is the average transfer interval length.

The system remains stable for τ around 0.06 s. The theoretical bound of τ_m on the linearized system from Theorem 2 is around 10^{-5} s. This discrepancy is due to the conservative nature of the Bellman–Gronwall Lemma. In Fig. 3, we show both a stable trajectory of the system, for $\tau = 0.08$ s, and an unstable trajectory for $\tau = 0.12$ s.

The transmission intervals in an NCS were modeled as random variables because of the effect of bursty traffic on the network. In our former simulation, the access to the network was modeled as the Poisson process. Two other models are proposed. The first, which we refer to as the spiked Poisson,

$$\|g(x, \hat{x})\| \le \|D\| \cdot \|e_x(t)\| \le \frac{\|D\| \cdot \|A_{11}\| \cdot \|A_{11} + D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\tau_m} - 1\right) \cdot e^{\|A_{11}+D\|\tau_m}}{1 - \|D\| \cdot \|A_{11} + D\|^{-1} \cdot \left(e^{\|A_{11}+D\|\tau_m} - 1\right)} \cdot \|x(t)\| = \gamma\|x(t)\|$$

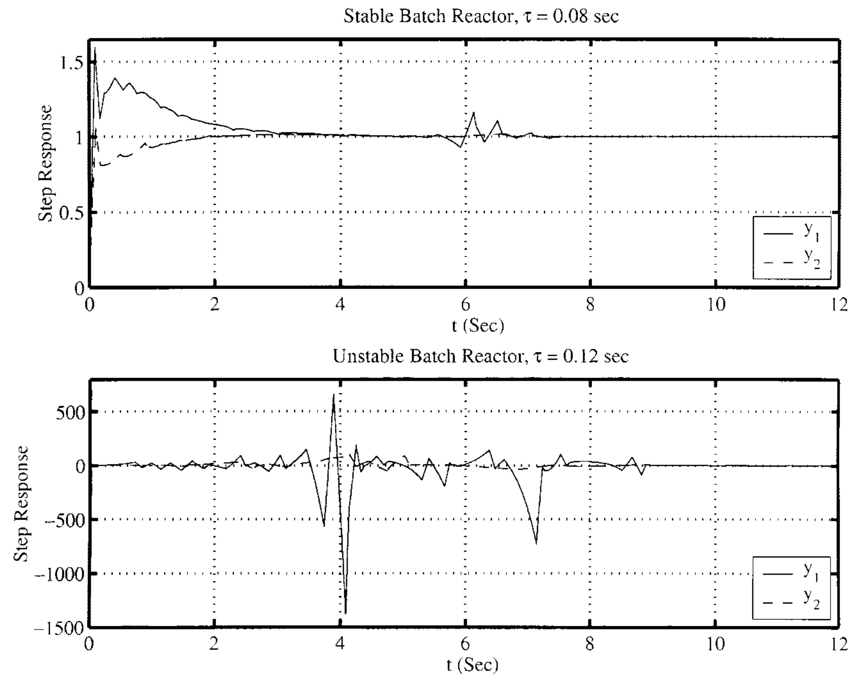

Fig. 3. Resulting trajectories of the batch reactor system with one-packet transmission. Note the different magnitude scales.

disallows any packets arrivals before $\tau/2$, and places half of the transmission interval times at exactly τ, the others, spread like a Poisson process. This emulates the controllers receiving access half of the time after a precise time interval, and on occasion being interrupted by other traffic on the network. Arbitrarily long delays are possible. The second model, which we refer to as the MATI (maximum allowable transfer interval) model, disallows arbitrarily long delays. Some mechanism in the network ensures that a deadline of $8\tau/7$ is always met. Such a model is more consistent with the theorems.

Fig. 4 compares the three different packet arrival models. Monte Carlo simulation is used because of the random nature of the network traffic. For each τ, a number of simulations were run and for each we checked if the control specifications (such as overshoot, rise time and settling time) were met. A constant transfer interval simulation was also run as a point of comparison. Notice that using the popular constant transfer interval (constant delay) network model would delude the control system designer into believing he or she need less network bandwidth than is actually required to meet control specifications. Of the three probabilistic models chosen, the plain Poisson arrival model shows the worst behavior, while the spiked Poisson and the MATI model are of comparable performance, close to but more conservative than the constant delay model. The simulation results reveal that the theoretical bound, on the order of tens of nanoseconds, is conservative since with average transfer interval $\tau < 50$ ms, all simulation results pass the test.

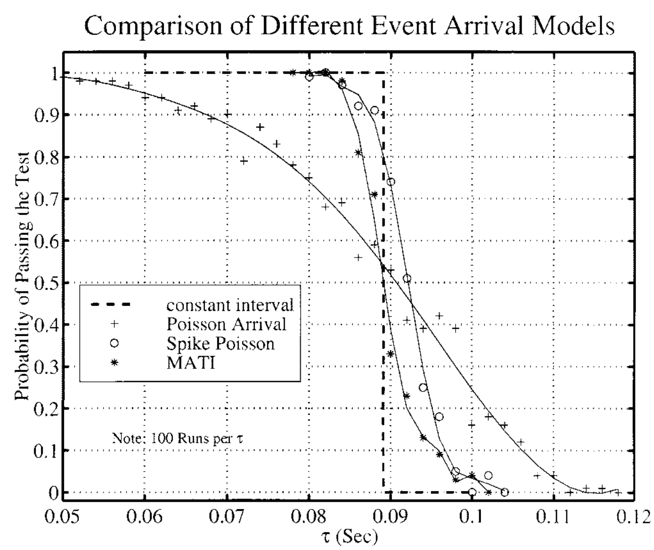

Fig. 4. Comparison of differing packet arrival models. Plant: Batch reactor, protocol: TOD.

B. Automotive Gas Turbine

The two-shaft automotive gas turbine is basically a coupled two-input two-output system [15, p. 249]. The two system outputs to be controlled are gas generator speed and inlet-turbine temperature. The two input variables are fuel pump excitation and nozzle actuator excitation. The linearized plant model $G(s)$ is shown in the equation at the bottom of the next page. The controller is designed in advance to reduce cross-channel interaction (or to realize "diagonal dominant"), to remove steady-state error and increase the system response speed without considering the network effects

$$K(s) = \frac{s+1}{s}\begin{bmatrix} 0.361 & 0.450 \\ -1.130 & 1.00 \end{bmatrix}.$$

The controller for a two-shaft gas turbine has many functions, for example, engine health monitoring (EHM) is currently of great interest. In the simulation, we show that the performance

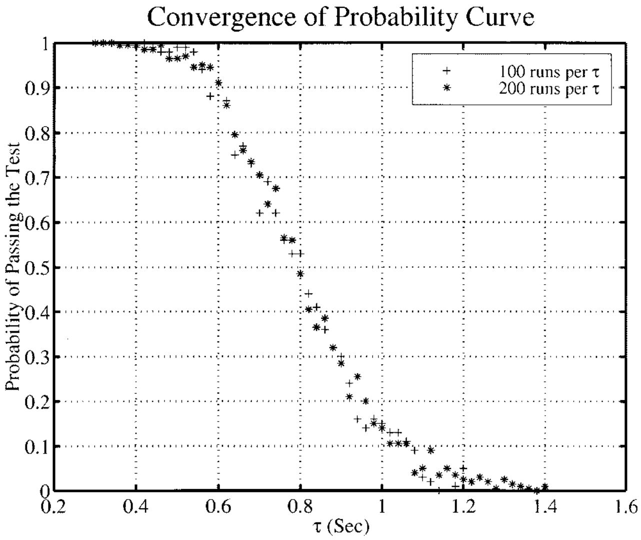

Fig. 5. Each point in this figure is the ratio of the number of simulations meeting the control specifications over the total number of simulations for fixed τ. The total number of simulations was either 100 or 200, and the difference between these resulting data points allows us to quantify our confidence in the data obtained using 100 simulations.

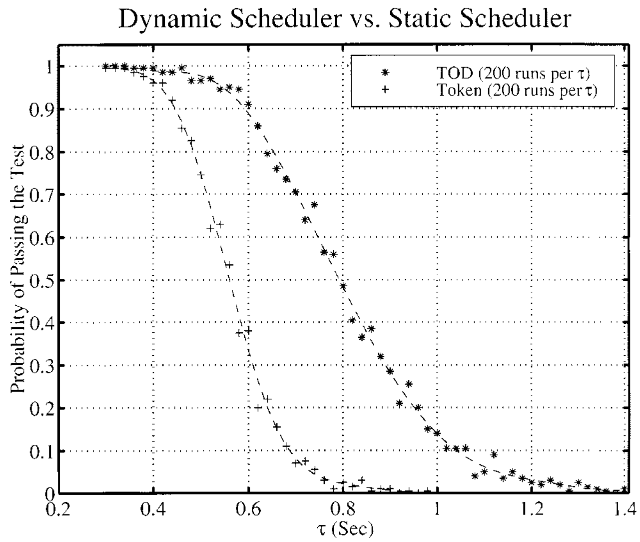

Fig. 6. The difference between a dynamic scheduler (using TOD) and a static scheduler (using a token ring) is shown. This simulation illustrates the advantage of our new TOD scheduling algorithm over the popular static scheduling algorithm.

of the steady-state operation control system is maintained when it is closed via a serial network, which is shared with other monitor and alarm nodes (like the flame detector and the lubrication monitor).

A study similar to that of the batch reactor was conducted. For each τ, a number of simulations were run and for each we checked if the control specifications were met. Fig. 5 shows the simulation results for the same system using the TOD scheduling algorithm. The difference is in the number of simulations used to approximate the probability of success. The graph gives us reason to be confident in a data point obtained even after only 100 simulations.

Two hundred simulations were used for each point to generate Fig. 6, which compares the performance of the dynamic TOD scheduler and the static scheduler. With enough effort, a static scheduling plan matching the performance of the dynamic scheduler can be found, but the best static schedule would most likely depend on initial conditions and perturbations. Compared with the simulation results (all runs pass the test when $\tau < 300$ milliseconds), Theorem 1 again returns a bound in the nanosecond range, because it is worst-case analysis and suitable for any kind of traffic distribution.

V. Conclusion

This paper makes three primary contributions. Foremost, it introduces in the form of the TOD protocol the concept of dynamically allocating network resources to those information sources with critical information. Second, it provides for the first time an analytic proof of stability for both the new protocol and the more common statically scheduled protocols. Finally, a proof of stability for the noncompetitive single packet case is given.

There are many important questions yet to be answered about the design of NCS. For example, the bounds provided by the theorems are based on perturbation theory and are shown to be conservative in the simulation studies. Finding tighter bounds on the required network speed is an area of great interest to the system designer. Also of importance to the system designer is the relative weights of errors between channels. Finding the best set of relative weights is an important question. In addition, we assume continuous priority levels as CAN II uses 29 bits in the identifier. A system designer may wish to allocate only a small subset of these bits to the competition, so generating the best mapping from error to discrete identifier (priority level) is also a subject of further research. Furthermore, output smoothing (or filtering) is another important application problem since the TOD protocol is more sensitive to sensor noise than static scheduler.

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$$\begin{bmatrix} \dfrac{0.806s + 0.264}{s^2 + 1.15s + 0.202} & \dfrac{-(15.0s + 1.42)}{s^3 + 12.8s^2 + 13.6s + 2.36} \\ \dfrac{1.95s^2 + 2.12s + 0.49}{s^3 + 9.15s^2 + 9.39s + 1.62} & \dfrac{7.14s^2 + 25.8s + 9.35}{s^4 + 20.8s^3 + 116.4s^2 + 111.6s + 18.8} \end{bmatrix}.$$

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