THE UNIVERSITY OF TEXAS AT SAN ANTONIO
EE 3413
ANALYSIS AND DESIGN OF CONTROL SYSTEMS

HOMEWORK # 5 SOLUTIONS
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The objective of this homework is to test your understanding of the content of Module 5. Due date of the homework is: **Thursday, February 25th, 2016, @ noon**.

You have to upload a scanned version of your solutions on Blackboard. If you don't have a scanner around you, you can use Cam Scanner—a mobile app that scans images in a neat way, as if they're scanned through a copier. Here's the link for Cam Scanner: https://www.camscanner.com/user/download.

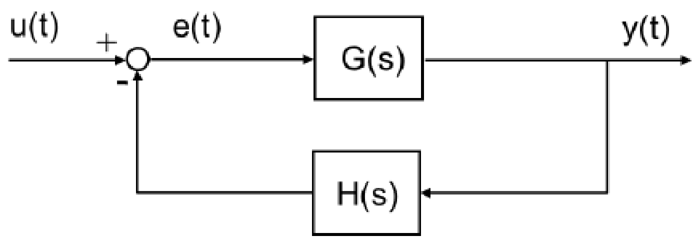

Figure 1: Feedback control system.

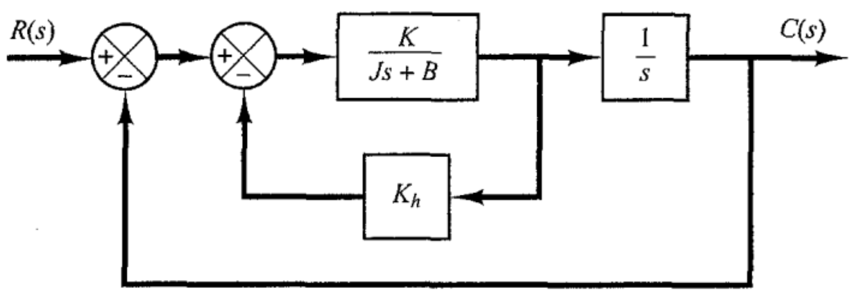

Figure 2: Servo system.

1. For a standard second order system given by this transfer function:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2},$$

where $\zeta = 0.6$ and $\omega_n = 5$. Answer the following questions.

(a) Find the: rise time, peak time, maximum overshoot, and settling time (the two criterion we discussed in class) if the system input is a unit step function.

(b) Show a plot of how t_r, t_p, and M_p all vary with respect to different values of ζ and ω_n. Ideally, you should do that on MATLAB.

Solutions (from Ogata):

From the given values of ζ and ω_n, we obtain $\omega_d = \omega_n\sqrt{1 - \zeta^2} = 4$ and $\sigma = \zeta\omega_n = 3$.

Rise time t_r: The rise time is

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - \beta}{4}$$

where β is given by

$$\beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1}\frac{4}{3} = 0.93 \text{ rad}$$

The rise time t_r is thus

$$t_r = \frac{3.14 - 0.93}{4} = 0.55 \text{ sec}$$

Peak time t_p: The peak time is

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = 0.785 \text{ sec}$$

Maximum overshoot M_p: The maximum overshoot is

$$M_p = e^{-(\sigma/\omega_d)\pi} = e^{-(3/4)\times 3.14} = 0.095$$

The maximum percent overshoot is thus 9.5%.

Settling time t_s: For the 2% criterion, the settling time is

$$t_s = \frac{4}{\sigma} = \frac{4}{3} = 1.33 \text{ sec}$$

For the 5% criterion,

$$t_s = \frac{3}{\sigma} = \frac{3}{3} = 1 \text{ sec}$$

2. For the system shown in Figure 1, assume that $G(s) = \dfrac{-K}{s+10}$ and $H(s) = 1$. Answer the following questions:

   (a) Find the closed-loop transfer function $Y(s)/U(s)$ and its pole (or poles).

   (b) What is the range of the constant K so that the closed-loop system is stable?

   (c) Suppose K = 5. What is the time constant of the closed-loop transfer function (as a first order system)?

   (d) What is the steady-state tracking error $e(\infty) = u(\infty) - y(\infty)$ under the input a unit step input $u(t)$?

   Solutions (from Ogata):

   (a) $\dfrac{Y(s)}{U(s)} = -\dfrac{K}{s-(K-10)}$

   (b) The pole of the system is $p = K - 10$. Hence, we need $p < 0$, then $K < 10$.

   (c) For $K = 5$, the transfer function can be written as:

   $$\frac{-1}{0.2s+1}.$$

   Hence, the time-constant is $T = 0.2$.

   (d) Under a unit step input, the steady state error $e(\infty) = u(\infty) - y(\infty) = 1 - (-1) = 2$.

3. For the system given in Figure 2, answer the following questions.

   (a) Obtain the transfer function $C(s)/R(s)$ in terms of constants K, J, B, K_h, and then write this system as a standard second order system as the transfer function given in Problem 1.

   (b) Determine the values of gain K and K_h so that M_p (the maximum overshoot) for a unit step response is equal to 0.2, and t_p (the peak time) is 1 second. Assume that $J = 1$ and $B = 1$.

   (c) With the above, now-obtained values for K and K_h, obtain the rise-time and settling time.

Solutions (from Ogata):

(a) $\dfrac{C(s)}{R(s)} = \dfrac{K}{Js^2 + (B + KK_h)s + K}$.

(b) The damping coefficient is $\zeta = \dfrac{B + KK_h}{2\sqrt{KJ}}$; natural frequency is $\omega_n = 2\sqrt{KJ}$. The maximum overshoot M_p is given by

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}$$

This value must be 0.2. Thus,

$$e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2$$

or

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61$$

which yields

$$\zeta = 0.456$$

The peak time t_p is specified as 1 sec; therefore, from Equation (5–20),

$$t_p = \frac{\pi}{\omega_d} = 1$$

or

$$\omega_d = 3.14$$

Since ζ is 0.456, ω_n is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 3.53$$

Since the natural frequency ω_n is equal to $\sqrt{K/J}$,

$$K = J\omega_n^2 = \omega_n^2 = 12.5 \text{ N-m}$$

Then K_h is, from Equation (5–25),

$$K_h = \frac{2\sqrt{KJ}\zeta - B}{K} = \frac{2\sqrt{K}\zeta - 1}{K} = 0.178 \text{ sec}$$

Rise time t_r: From Equation (5–19), the rise time t_r is

Rise time t_r: From Equation (5–19), the rise time t_r is

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where

$$\beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1} 1.95 = 1.10$$

Thus, t_r is

$$t_r = 0.65 \text{ sec}$$

Settling time t_s: For the 2% criterion,

$$t_s = \frac{4}{\sigma} = 2.48 \text{ sec}$$

For the 5% criterion,

$$t_s = \frac{3}{\sigma} = 1.86 \text{ sec}$$

(c)