QUIZ # 1

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Given the following LTI dynamical system:

$$\dot{\mathbf{x}}(t) = \boldsymbol{A}\mathbf{x}(t) + \boldsymbol{B}\mathbf{u}(t), \quad \mathbf{x}_{\text{initial}} = \mathbf{x}_{t_0} \tag{1}$$

where:

- $\mathbf{x}(t)$: dynamic state-vector of the LTI system, $\mathbf{u}(t)$: control input-vector
- $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are constant matrices.

The closed-form to the above differential equation for any time-varying control input is given by:

$$\mathbf{x}(t) = e^{\boldsymbol{A}(t-t_0)}\mathbf{x}_{t_0} + \int_{t_0}^{t} e^{\boldsymbol{A}(t-\tau)}\boldsymbol{B}\mathbf{u}(\tau)\, d\tau.$$

Show that the above solution is in fact a solution to the system dynamics in (1).
Hint — Leibniz Differentiation Theorem:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\int_{a(\theta)}^{b(\theta)} f(x,\theta)\,\mathrm{d}x\right) = \int_{a(\theta)}^{b(\theta)} \partial_\theta f(x,\theta)\,\mathrm{d}x \,+\, f\big(b(\theta),\theta\big)\cdot b'(\theta) \,-\, f\big(a(\theta),\theta\big)\cdot a'(\theta)$$

Your Solution: