A function $\phi(x, u)$ is Globally Lipschitz (*Lipschitz Continuous*) with Lipschitz constant L if and only if:

$$\|\boldsymbol{\phi}(x_1, u) - \boldsymbol{\phi}(x_2, u)\| \leq L\|x_1 - x_2\|, \quad L \geq 0.$$

Find the Lipschitz constant for the following functions:

1. $\phi(x) = x^4$, if $x \in [-2, 2]$. You will have to use the triangular inequality.

   Hint 1: $b^4 - a^4 = (b - a)(b^3 + b^2a + ba^2 + a^3)$

2. $\phi(y, x) = \sqrt{y^2 + x^2}$, with $x \in [-1, 1]$. You should apply the definition on y here.

   Hint 2: You will have to multiply by a fraction that allows you to use

   $$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a^2 - b^2$$

   Hint 3: Also, don't forget that $|a^2 - b^2| = |a - b||a + b|$.