EE3413

Quiz #2 Solution

$$f(t) = 2te^{7t} + \overbrace{4\cos(25)}^{\text{constant}}$$

$$= 2te^{7t} + 3.62$$

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt.$$

$$F(s) = \int_0^\infty e^{-st}\left(2te^{7t} + 3.62\right)dt = \underbrace{\int_0^\infty 2te^{(7-s)t}\,dt}_{F_1(s)} + \underbrace{\int_0^\infty 3.62\,e^{-st}\,dt}_{F_2(s)}$$

$$F_1(s) = \int_0^\infty 2te^{(7-s)t}\,dt = 2\int_0^\infty te^{-(s-7)t}\,dt$$

$\xrightarrow{\text{integration by parts}}$ $\int uv' = uv - \int u'v$

$$u = t \qquad v' = dv = e^{-(s-7)t}$$

$$du = dt \qquad v = -\frac{1}{(s-7)}e^{-(s-7)t}$$

$$= 2\left(-\frac{t}{(s-7)}e^{-(s-7)t} - \int_0^\infty -\frac{1}{(s-7)}e^{-(s-7)t}\,dt\right)$$

$$= 2\left[-\frac{t}{(s-7)}e^{-(s-7)t} - \left(\frac{1}{(s-7)^2}e^{-(s-7)t}\Big|_0^\infty\right)\right] = 2\left[\underbrace{-\frac{t}{(s-7)}e^{-(s-7)t}}_{0} - 0 + \frac{1}{(s-7)^2}\right] = \frac{2}{(s-7)^2}$$

$$F_2(s) = \int_0^\infty 3.62\,e^{-st}\,dt = 3.62\left(-\frac{1}{s}e^{-st}\Big|_0^\infty\right) = \frac{3.62}{s}$$

$$F(s) = F_1(s) + F_2(s) = \boxed{\frac{2}{(s-7)^2} + \frac{3.62}{s}}$$