QUIZ # 8
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Solve the following problems, given the augmented MPC dynamics,

$$
\begin{aligned}
x_a(k+1) &= \Phi_a x_a(k) + \Gamma_a \Delta u(k) \\
y(k) &= C_a x_a(k), \;\; x_a \in \mathbb{R}^{n+p}, \Gamma_a \in \mathbb{R}^{n+p\times m}, C_a \in \mathbb{R}^{p\times n+p}.
\end{aligned}
$$

1. For a predicted horizon N_p, derive an equation that relates the predicted outputs to $x_a(k)$ and the MPC variables Δu. That is, derive matrices W, Z in this equation:

$$
Y = \begin{bmatrix} y(k+1|k) \\ y(k+2|k) \\ \dots \\ y(k+N_p|k) \end{bmatrix} = W x_a(k) + Z \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_p-1) \end{bmatrix} = W x_a(k) + Z \Delta U.
$$

2. Derive the optimal ΔU^* if the given cost function is (without constraints):

$$J(\Delta U) = \frac{1}{2}(r - Y)^\top Q(r - Y) + \frac{1}{2}\Delta U^\top R \Delta U, \;\; Q = Q^\top \succ 0, R = R^T \succ 0.$$

3. Suppose that you're given the following constraints on the rate of change of the control action:

$$u^{\min} \leq \Delta U \leq u^{\max}.$$

   Write the corresponding optimization problem in the following form:

$$\begin{array}{ll} \text{minimize} & J(\Delta U) \\ \text{subject to} & g(\Delta U) \leq 0 \; , \end{array}$$

   where $g(\Delta U)$ is a linear set of constraints.