QUIZ # 9
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Given the following plant dynamics:

$$\dot{x}_p = A_p x_p + B_p^{(1)} u_1 + B_p^{(2)} u_2$$
$$y = C_p x_p, \;\; x_p(0) \;\text{ not given}$$

where $u_2(t)$ is the unknown input vector. The system consists of n states, m_1 known inputs, m_2 unknown inputs, and p measurable outputs. We want to design a dynamic unknown input observer (UIO) which takes the following form:

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1, \\ \hat{x}_p &= x_c + My, \end{aligned}$$

The UIO is motivated by writing x_p as:

$$x_p = (I - MC_p)x_p + MC_p x_p = x_c + My.$$

1. Assume that the updated x_c takes the following form:

$$x_c = (I - MC_p)x_p.$$

   (a) Find $\dot{x}_c = A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1$,, where $A_c, B_c^{(1)}, B_c^{(2)}$ are matrices that you should determine, assuming that the unknown input vector is nullified and a convergence term is added to x_c, as discussed in class. Note that $\hat{x}_p = x_c + My$;

   (b) Derive the matrix equality that guarantees the nullification of $u_2(t)$.

   Precisely, you should find $A_c, B_c^{(1)}, B_c^{(2)}$ in terms of $A_p, B_p^{(1)}, C_p, M, L$.

2. If $p = m = n$, and $C_p B_p^{(2)}$ is invertible (and obviously square), what is a closed-form solution for the design matrix M?

3. Derive the estimation error dynamics.